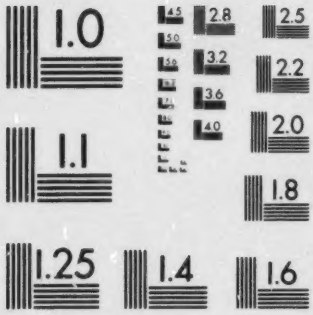

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ABSTRACT

This curriculum support document, developed by a committee of teachers in Manitoba, contains information and practical applications dealing with key concepts of problem solving in mathematics for grades 7, 8, and 9. Contained in the handbook are: (1) an introduction that establishes the background and identifies the goal and objectives of the handbook; (2) a general overview of problem solving that describes key strategies with appropriate examples and provides suggestions on teaching techniques; (3) problem-solving sheets, grouped into sections by grade level and including answers and selected solutions, with a curriculum chart preceding each section; and (4) a bibliography of problem-solving resources. (MNS)

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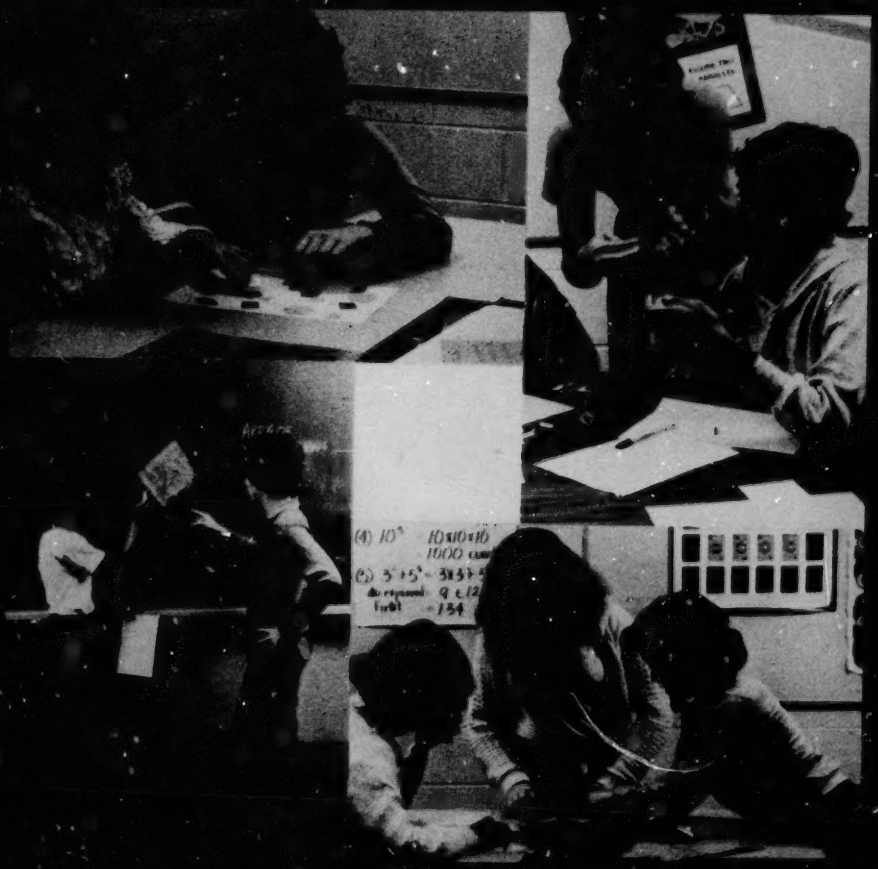

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PROBLEM SOLVING HANDBOOK

FOR

GRADES 7, 8, 9 MATHEMATICS

1982

PROVINCE OF MANITOBA

DEPARTMENT OF EDUCATION

ISBN 0-86497-104-4

FOREWORD

This curriculum support document contains information and practical applications which deal with the key concepts of problem solving in mathematics. It has been prepared for teachers to use in their teaching programs. The contents reflect the most recent research findings on problem solving.

Contained in the handbook are

- an introduction that establishes the background and identifies the goal and objectives of the handbook;
- a general overview of problem solving that describes key strategies with appropriate examples and provides suggestions on teaching techniques;
- problem solving sheets, grouped by grade level, that include answers and selected solutions; each section is preceeded by a curriculum chart;
- a bibliography of problem-solving resources.

The content of this handbook has been selected for practical use by teachers in their classrooms. It is highly recommended to all teachers of grades 7, 8 and 9 mathematics.

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ACKNOWLEDGEMENTS

The Department of Education acknowledges with appreciation the contribution of all those who assisted in the production of the Problem Solving Handbook for Grades 7 - 9 Mathematics.

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The Department would also like to thank the staff and students of the following schools for their cooperation in the taking of the photographs that appear on the front cover:

Elmwood High School, Winnipeg School Division No. 1

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INTRODUCTION

In "An Agenda For Action: Recommendations For School Mathematics of the 1980's", the National Council of Teachers of Mathematics states that problem solving should be the focus of school mathematics programs.

Special recommendations are made to support their statements. Some of them are:

1. Educators should give priority to the identification and analysis of specific problem solving strategies.
2. Educators should develop and disseminate examples of "good problems" and strategies and suggest the scope of problem solving activities for each school level.
3. Appropriate curricular materials to teach problem solving should be developed for all grade levels.
4. Mathematics programs should involve students in problem solving by presenting applications at all grade levels.

Subsequent to this report, the Manitoba Association of Mathematics Teachers and the Department of Education held a three-day seminar for representative teachers of all school divisions in Manitoba. The threefold purpose of the seminar was to identify the key strategies for problem solving at the Primary, Intermediate and Junior High levels; to establish typical problems and practical applications at each level for student use; to prepare teachers to conduct divisional in-services on problem solving.

Following the seminar, over thirty problem solving in-services were presented by the seminar teachers throughout the province with consultative and financial support from the Department of Education. The teachers who participated in the in-services expressed a need for more support materials in problem solving which would interface with the Manitoba Mathematics Curriculum Guide.

In 1981, in response to the expressed need, the Department of Education authorized the setting up of a committee of teachers to develop a problem solving handbook for grades 7, 8 and 9 mathematics. The Mathematics Working Party formulated the criteria for the development of the handbook. Using this criteria the goals and the objectives were established.

Goal of the Handbook

The goal of the handbook is to provide ideas and resources that will help teachers to teach problem solving more effectively.

Specific Objectives

The specific objectives of the handbook are:

1. to develop a better understanding of problem solving;
2. to develop an understanding of useful strategies and their applications to problem solving;
3. to integrate problem solving into the regular mathematics program by using a collection of problem sheets which support the curriculum topics;
4. to stimulate critical thinking and creativity in problem solving;
5. to provide incentive for creating classroom environments in which problem solving can flourish.

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overview of problem solving

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OVERVIEW OF PROBLEM SOLVING

Mathematical Problem Defined

A mathematical problem exists when the following conditions are met:

1. The situation contains a given state and a goal state.
2. One of two conditions exist:
   a) the solution which leads to the goal state is readily apparent.
   b) there is a blockage between the given state and the goal state.
3. The situation involves mathematics.
4. There is a desire to find a solution to the problem.

Problem Types

There are two types of problems. In the first type, the solution comes almost immediately and without deliberation. In this case, the problem solver perceives a solution pattern as a result of previously acquired knowledge and past experiences.

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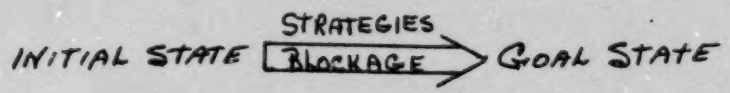
In the second type, the solution does not seem so immediately obvious, and the problem solver must search for a means of solving the problem. By applying one or more strategies, he or she can successfully find the solution.



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Problem Solving Strategies

Strategies are skills which can be used to solve dissimilar problems. These skills have to be learned and understood through practical application. The purpose of this section is to identify and describe the key problem solving strategies and to provide examples of problems for each strategy.

The strategies are:

- Search for a Pattern
- Classify Information
- Account for All Possibilities
- Experiment
- Guess and Check
- Work Backwards
- Make a Diagram or Graph
- Write an Open Sentence or Use a Formula
- Partition
- Solve a Simpler Problem
- Check for Hidden Assumptions
- Logic

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The strategy descriptions on the following pages each contain two problem examples and their solutions. It should be noted that other strategies could have been applied in the solution of the problems.

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STRATEGY: Search for a Pattern

Many problems may be solved by recognizing a pattern. In some instances an active search for a pattern will involve such activities as grouping, regrouping, reorganizing the data and modifying certain conditions.

Example 1: *Find the next two numbers in the sequence 3, 6, 11, 18, ...*

Solution: Let us look at the differences between each consecutive pair of numbers in the sequence.

$$3,\ \underset{3}{\smile}\ 6,\ \underset{5}{\smile}\ 11,\ \underset{7}{\smile}\ 18,\ \underset{9}{\smile}\ \_,\ \underset{11}{\smile}\ \_,\ \ldots$$

The difference between the first two numbers is 3, the next two numbers is 5, and the next two numbers is 7. Continuing the pattern, the differences between the next two pairs of numbers are 9 and 11. Therefore, the required numbers are 27, and 38 (i.e., 18 + 9 = 27 and + 11 = 38).

Example 2: *Find the sum of 1 - 2 + 3 - 4 + 5 - 6 ... + 99 - 100.*

Solution: Let us group the first two numbers, the next two numbers, and so on.

$$(1-2) + (3-4) + (5-6) + \ldots + (99-100).$$

Since the sum of each pair of numbers is -1 and there are 50 pairs from 1 to 100, then the sum is 50 (-1) = -50.

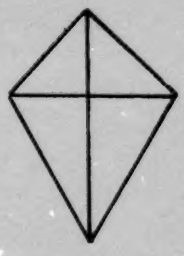
STRATEGY: Classify Information

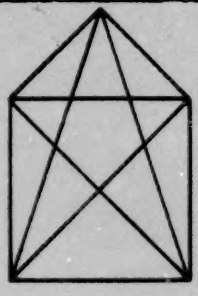
Classifying information in a table or chart helps to discover a pattern more easily, to identify missing data, and to eliminate superfluous information.

Example 1: *A diagonal is a line segment joining two vertices that are not end points of the same side. Compute the number of diagonals of a decagon (10-sided polygon). The polygon need not be regular.*

Solution: Make a table.

Number of sides of each polygon	3	4	5	6	7	8	9	10
Number of diagonals in each polygon	0	2						





Count the number of diagonals in the pentagon. Construct a hexagon and draw all possible diagonals. List this information in the table. Use the table to discover the pattern for the number of diagonals in the remaining polygons. The number of diagonals of the decagon is 35.

Example 2: *What is the last digit of 2^{29}?*

Solution:

Power	Exponent	Answer	Last Digit
2^1	1	2	2
2^2	2	4	4
2^3	3	8	8
2^4	4	16	6
2^5	5	32	2
2^6	6	64	4
2^7	7	128	8
2^8	8	256	6

The last digit repeats itself every fourth time. Since 29 gives a remainder of 1 when divided by 4, then 2^{29} ends in a 2.

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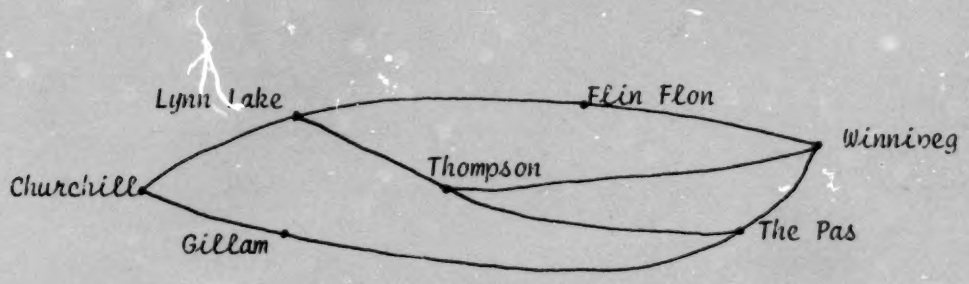
STRATEGY: Account for All Possibilities

Identifying all possibilities, especially in a systematic way, accounts for and verifies all possible solutions.

Example 1: *David, who lives in Churchill, is preparing to visit his uncle in Winnipeg. Using the most direct flight routes, he made the following diagram.*



How many different direct routes can he fly to Winnipeg?

Solution:
1) Churchill - Lynn Lake - Flin Flon - Winnipeg
2) Churchill - Lynn Lake - Thompson - Winnipeg
3) Churchill - Lynn Lake - Thompson - The Pas - Winnipeg
4) Churchill - Gillam - The Pas - Winnipeg

Therefore, there are four possible direct routes.

Example 2: *How many numbers less than 150 and greater than 0 do not contain the digit 3?*

Solution: Let us find all numbers that contain a 3.
From 30 to 39, there are 10 numbers containing a 3.
From 130 to 139, there are 10 numbers containing a 3.
Starting at 3, every tenth number contains a 3. There are 15.
However, we have accounted for 33 and 133 twice.
Therefore, there are (10 + 10 + 15 - 2) = 33 numbers containing 3.
Then there are 149 - 33 = 116 numbers which do not contain a 3.

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STRATEGY: Experiment

The experimental process consists of manipulating concrete material, dramatizing a situation, or making a model. In an abstract problem, this strategy makes it easier to discover relationships.

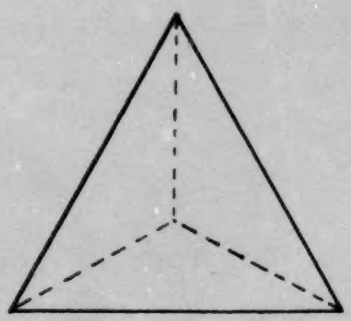
Example 1: *How can change for 75¢ be made using exactly 6 coins?*

Solution: Use coins or replicas. There are three solutions.

i) Three nickels, one dime and two quarters.
ii) Five nickels and one 50 cent piece.
iii) Five dimes and one quarter.

Example 2: *You are given six toothpicks of equal length. Show how they can be arranged to form four equilateral triangles.*

Solution: Manipulate six toothpicks to form equilateral triangles. The model is three-dimensional and will appear as follows.



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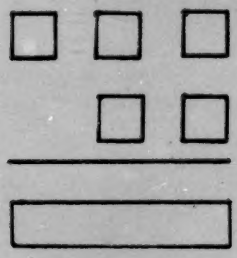
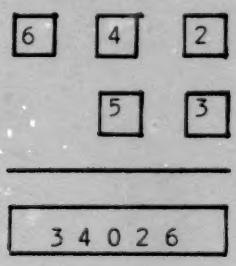
STRATEGY: Guess and Check

Using the given information, an educated guess is made and checked against the problem conditions. This process is repeated until a correct solution is found.

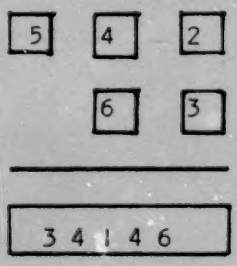
Example 1: *Arrange the numbers 2, 3, 4, 5, 6 in the boxes to get the largest possible product. Use each number once.*



Solution: Many possibilities may have to be tried to determine the largest product.



A TRIAL SOLUTION



FINAL SOLUTION

Example 2: *What digit must K represent if the number K434K0 is divisible by 36?*

Solution: Using various numbers, guess and check until 8 is tested.
ANSWER: K = 8 and the number is 843480.

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STRATEGY: Work Backwards

Start with the end result and work backwards toward the initial condition to find a solution.

Example 1: *Susan delivers papers every day except Sunday. Every Saturday she collects $24.00. If each paper costs 25¢, how many customers does she have?*

Solution: Working backwards, divide $24.00 by 25¢ to find the number of papers delivered and then divide by 6 (6 day week) to find the number of customers.

The number of papers is: $24.00 \div 0.25 = 96$

The number of customers is: $96 \div 6 = 16$

Example 2: *A field has a perimeter of 300m. If the length of the field is twice the width, find the dimensions of the field.*

Solution: Use the given perimeter and the perimeter formula. Work backwards by filling in and testing possible lengths and widths to determine the solution.

$P = 300m$

$P = 2l + 2w$

$P = 2(?) + 2(?)$

$P = 2(100) + 2(50)$

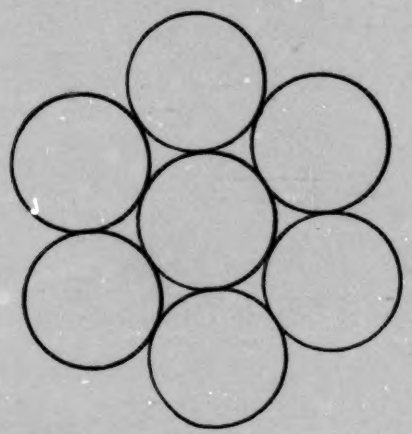
Therefore, $l = 100m$ and $w = 50m$.

STRATEGY: Make a Diagram or Graph

If the given information in a problem is diagrammed or graphed, relationships are visualized more easily and help to determine a solution.

Example 1: *How many quarters can be placed around a given quarter so that all adjacent quarters touch the given quarter?*

Solution: Make a diagram using a quarter or facsimile.

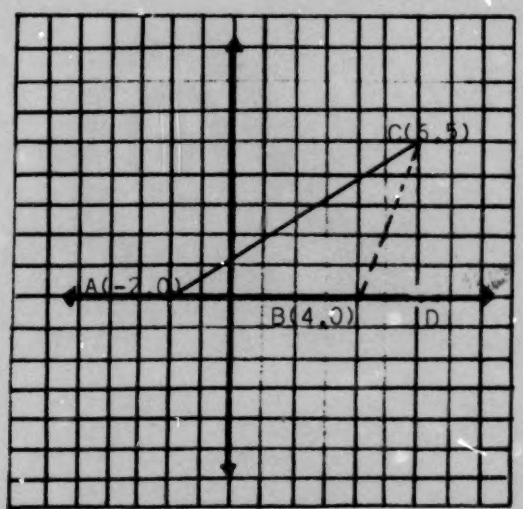


Count the number of quarters surrounding the centre quarter.

ANSWER: 6

Example 2: *Find the area of a triangle whose vertices are A(-2, 0), B(4,0) and C(6,5).*

Solution:



Base of triangle: AB = 6

Height of triangle: CD = 5

Area of $\triangle ABC = \frac{(AB)(CD)}{2}$

$= \frac{(6)(5)}{2}$

$= 15$

The area is 15 square units.

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STRATEGY: Write an Open Sentence or Use a Formula

The given information and its wording frequently suggest the use of an open sentence (equation). Other strategies are often used to set up the open sentence, such as making diagrams or restating the problem. Once the open sentence has been written, the main portion of the problem solving process has been completed.

Example 1: *Jean is thinking of a number. If she increases it by 2 and then divides it by 3, the result will be the original number. What is the original number?*

Solution: Let n be the number.

Then $\frac{n + 2}{3} = n$

$n + 2 = 3n$

$2 = 3n - n$

$2 = 2n$

$1 = n$

Therefore, the number is 1.

Example 2: *Pierre wants to drive a distance of 2 km at the average rate of 60 km/h. If he drives the first km at 30 km/h, how fast must he drive the second km to average 60 km/h?*

Solution: Use the formula, Time = $\frac{\text{Distance}}{\text{Speed}}$ to find the time required to travel 2 km at the speed of 60 km/h.

i.e. $t = \frac{d}{s}$

$t = \frac{2}{60}$

$t = \frac{1}{30}$ hour

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The number of minutes is $\frac{1}{30} \times 60 = 2$

However, the first km was driven at 30 km/h. The time required to drive the first km is:

$$t = \frac{d}{s}$$

$$t = \frac{1}{30} \text{ hour}$$

The number of minutes is: $\frac{1}{30} \times 60 = 2$

Since the remaining time to drive the second km is zero, it is impossible to make up for the lost time and drive fast enough to raise the average speed to 60 km/h.

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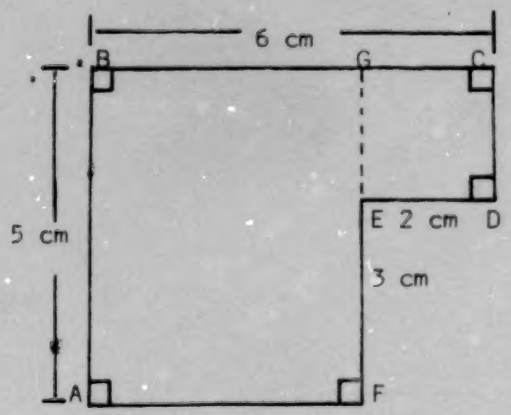
STRATEGY: Partition

Solutions to some problems require a series of actions, each one dependent on the other. By dividing the problem into smaller parts where each part has an obvious solution, the necessary steps are established leading to the final solution of the given problem.

Example 1: *Calculate the area of the given figure.*



Solution: Partition the figure by extending $\overline{FE}$ to G.
Now calculate the area by adding together the areas of the two rectangles.

Total Area = (Area of ABGF) + (Area of EGCD)
= (5 x 4) + (2 x 2)
= 20 + 4
= 24 cm^2

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Example 2: *Complete the "Magic Square" below so that the sum in each direction is equal to 540.*

144		
324		
		216

Solution: Solve this problem one step at a time - adding or subtracting as required.

Step 1: $540 - (144 + 324)$
$= 72$

Step 2: $540 - (216 + 72)$
$= 252$

Step 3: $540 - (216 + 144)$
$= 180$

Continue the steps to complete the solution as shown below.

144	108	288
324	180	36
72	252	216

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STRATEGY: Solve a Simpler Problem

To solve a complex problem, a similar problem that is less complex may be written and solved. This procedure will help to determine the solution strategy for the complex problem.

Example 1: *The State of Frustration has an area of 2 199 800.3 km^2. If the area per person is 11.35 km^2, what is the population of the State?*

Solution: Solve the problem by using smaller numbers. Assume the area is 10 km^2 and there are 2 km^2 per person. Then the population would be $\frac{10}{2}$ = 5 people.

Consequently, the answer to the original problem is

$$\frac{2\ 199\ 800.3}{11.35} = 193\ 815 \text{ people}$$

Example 2: *How thick is a sheet of paper? Find the answer using only a ruler as a measuring instrument.*

Solution: Solve the problem by using larger numbers. Since measuring the thickness of 1 sheet of paper is not practical, find the thickness of 100 sheets of paper and divide by 100.

STRATEGY: Check for Assumptions

Assumptions made about the information in the problem often hinder progress towards its solution. To bring these assumptions out into the open, the following questions can be asked:

"What precisely does the problem say?"
"What does it not say?"
"What am I assuming that is not implied or explicitly stated?"

Example 1: *Two explorers set out on foot to explore the barren area of the north. They plan to go in a straight line and return by the same route. Each can carry 12 days of food supplies and travel 20 kilometres a day. What is the farthest distance that can be explored?*

Solution: HIDDEN ASSUMPTION: They must travel together.
A and B start out with food for 12 days. At the end of 4 days they each have a supply for 8 days. B takes 4 days of his supply to use on the way back and gives A his supply for the other 4 days. A now has a supply for 12 days. He continues for 4 more days, and then returns over a period of 8 days. He will have explored 160 kilometres (8 days out and 8 days back).

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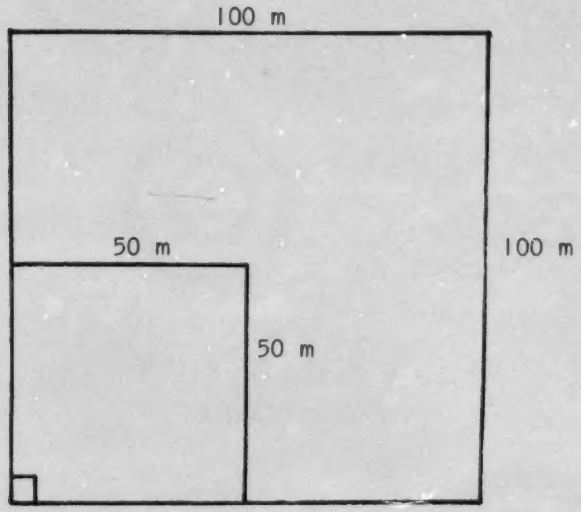
Example 2: *If a boy can mow a lawn 100 metres by 100 metres in 60 minutes, how long will it take him to mow a lawn 50 metres by 50 metres?*

Solution: HIDDEN ASSUMPTION: 50 x 50 is half the size of 100 x 100.



(50 x 50) is only $\frac{1}{4}$ the size of (100 x 100). Therefore, it will take the boy only $\frac{1}{4}$ as long or $\frac{1}{4}$ x 60 or 15 minutes.

STRATEGY: Logic

Logic is involved in most problem solving. In many cases the logic strategy involves the perception of relationships that are not readily apparent. Frequently, questions like "if this is false, then what follows?" are asked while applying the logic strategy.

Example 1: *A certain kind of bacteria doubles in quantity every hour. At midnight two nights ago, there were only two bacteria in the jar. At midnight last night, the jar was full. When was the jar half full?*

Solution: Since the bacteria doubles every hour, the jar must have been half full one hour previous to being full, i.e., at 11:00 p.m. last night.

Example 2: *There are eight baseballs, all exactly alike in size and appearance, but one is heavier than any of the other seven which are all the same weight. With a balance scale, how can the heaviest baseball be positively determined with only two weighings?*

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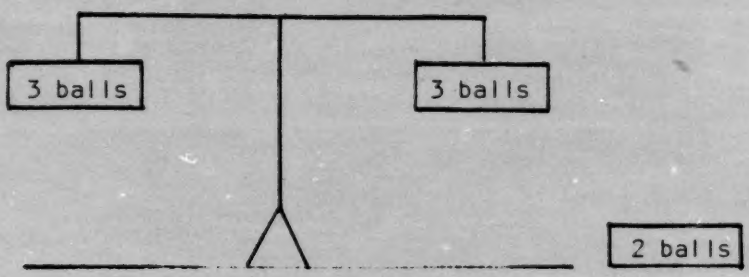
Solution: First Weighing:



Second Weighing:

If there is a balance in the first weighing, then the heavier ball must be one of the two balls set aside. To determine the heavier ball, compare the weights of the two balls by placing one on each pan.

If there is no balance in the first weighing, then the heavier group of three balls contains the heavier ball. To determine the heavier ball, place one ball from the heavier group of three on each pan and set one aside. If there is a balance, the heavier is the one set aside. If there is no balance, the heavier ball is on one of the pans.

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Suggested Teaching Techniques

The problem solving sheets in the next section are designed for use by students. Following are some suggestions which may be helpful to the teacher for using these problem solving sheets in the regular mathematics program.

There are two instructional techniques, amongst others, which may be used to teach problem solving to students:

1. the intuitive development of strategies when actually solving problems;
2. the formal development of problem solving strategies followed by their application to solving problems.

The experiences of some teachers who have integrated problem solving into their mathematics programs suggests that the intuitive development of problem solving strategies may be more effective.

The development of problem solving strategies may be better achieved by structuring the classroom in various ways which will encourage the active participation of all students. This can be done by having students work individually, in pairs, or in small groups.

Whatever technique is used, it is important that the problem solving program is integrated into the regular mathematics curriculum and that it be continuous and developmental. This can be achieved by teaching problem solving daily or by setting aside a unit of time, such as one period per cycle.

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How To Use the Problem Solving Sheets

The problem solving sheets may be used in their present form or adjusted to meet the classroom needs. Some possible uses or adjustments are:

1. refer to the appropriate curriculum chart located at the beginning of each grade to identify mathematical topics and possible strategies for each problem solving sheet;
2. add and/or replace problems to make the problem solving sheets more effective in meeting the needs of students;
3. change the order of the sheets to correlate with the curriculum topics being taught;
4. separate the problems on the sheets for use in bingo, tic tac toe or any other games;
5. encourage the use of estimation;
6. encourage the use of various strategies to solve a particular problem;
7. encourage the use of calculators when computation is cumbersome and detracts from the problem solving process;
8. use calculator problem solving sheets and superstar sheets as additional skill building and motivational activities;
9. use problem solving sheets from the previous grade when necessary to develop an understanding of the basic strategies.

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Evaluation of Problem Solving

The selection and the application of an appropriate strategy to solve a problem is important. Therefore evaluation methods should be flexible and varied to reflect the use of strategies in the solution of problems. The following suggestions may be used to evaluate student progress:

1. Evaluation should not only help to determine student progress but also promote a positive attitude in students towards problem solving.
2. Evaluation of problem solving should be integrated with the regular mathematics program.

3. The difficulty level of problems should be within the reach of the majority of students.
4. Students should be encouraged to identify in their own words the strategy they used and, where appropriate, show the work when solving a problem.
5. Student progress in problem solving should be evaluated continuously.
6. Student progress in problem solving may be evaluated through oral discussion and classroom activities.
7. Student progress in problem solving may be evaluated through written assignments.

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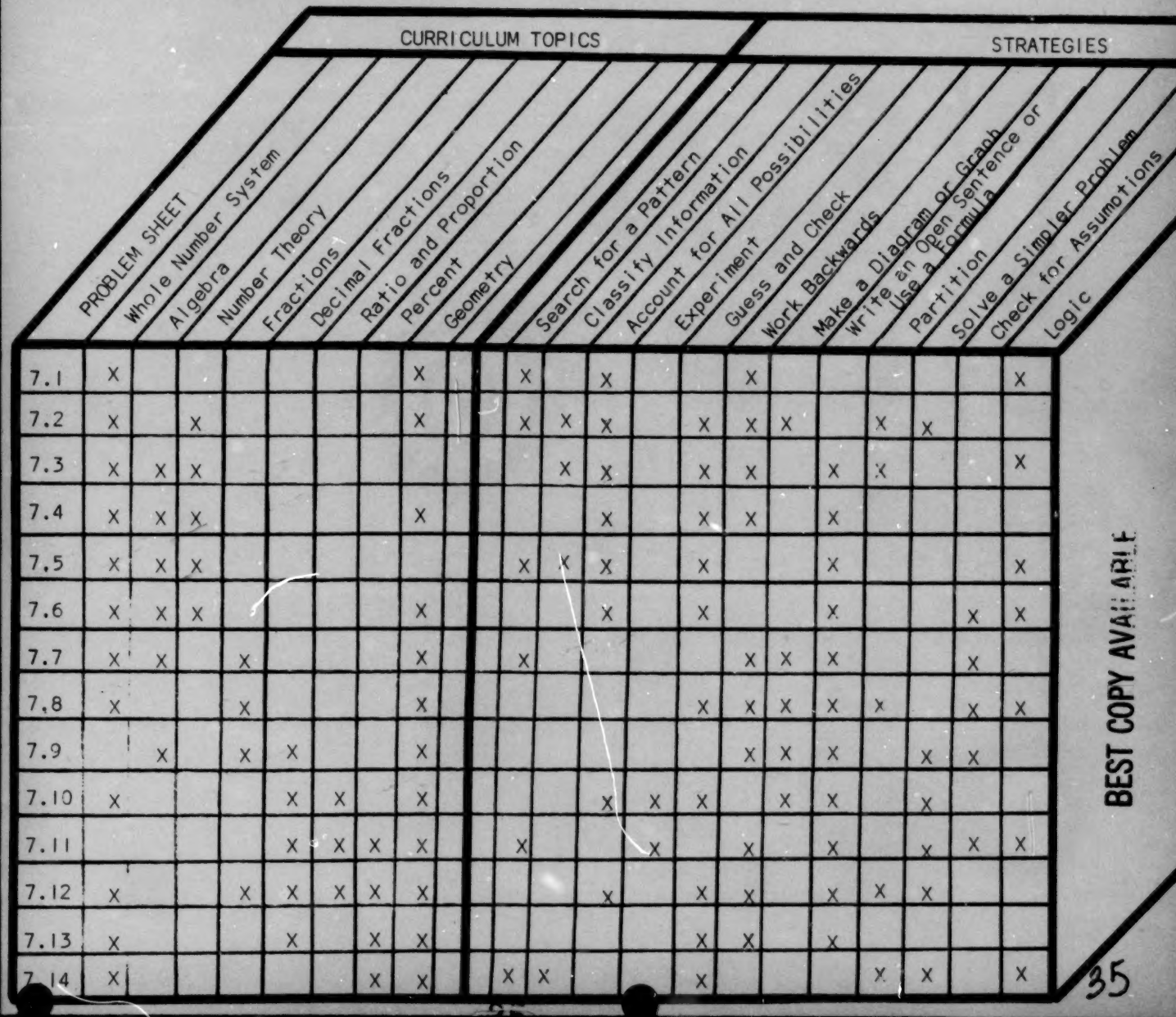
grade 7

problem sheets

(with answers and selected solutions)

CURRICULUM CHART: GRADE 7


CURRICULUM TOPICS
STRATEGIES
PROBLEM SHEET
Whole Number System
Algebra
Number Theory
Fractions
Decimal Fractions
Ratio and Proportion
Percent
Geometry
Search for a Pattern
Classify Information
Account for All Possibilities
Experiment
Guess and Check
Work Backwards
Make a Diagram or Graph
Write an Open Sentence or Use a Formula
Partition
Solve a Simpler Problem
Check for Assumptions
Logic
7.1 X X X X
7.2 X X X X X X X X X X X X
7.3 X X X X X X X X X X X X
7.4 X X X X X X X X X X
7.5 X X X X X X X X X X X X
7.6 X X X X X X X X X X X X
7.7 X X X X X X X X X X X
7.8 X X X X X X X X X X X X
7.9 X X X X X X X X X X X X
7.10 X X X X X X X X X X X
7.11 X X X X X X X X X X X X
7.12 X X X X X X X X X X X X
7.13 X X X X X X X X X X
7.14 X X X X X X X X X X X X

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PROBLEM SHEET 7.1

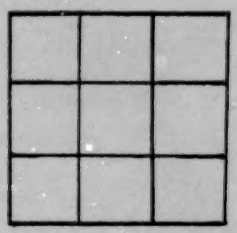
1. Find the missing numbers in this multiplication table.

X	3			6
9	27	45	72	
4				
		15		
	6			

2. Find the next three numbers in this sequence.

   1, 3, 6, 10, ___, ___, ___.

3. How many squares are in this figure?



4. In how many different ways is it possible to make change for fifty cents using quarters, dimes and nickels?

5. The sum of seven numbers is 2143. If each number is increased by 50 and the seven resulting numbers are added, what is the new sum?

6. Five boys wrote a mathematics test. The average mark was 68. If the marks of four boys were 75, 62, 84, and 53, what was the mark of the fifth boy?

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PROBLEM SHEET 7.1

ANSWERS AND SELECTED SOLUTIONS

1.

X	3	5	8	6
9	27	45	72	54
4	12	20	32	24
3	9	15	24	18
2	6	10	16	12

2. 15, 21, 28

3. 14 squares

4. 10 ways

5. $(7 \times 50 + 2143) = 2493$

6. $68 \times 5 - (75 + 62 + 84 + 53) = 66$

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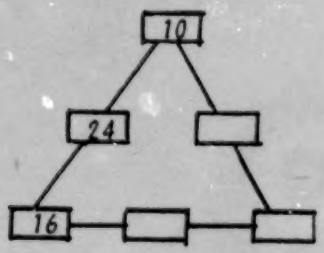
PROBLEM SHEET 7.2

1. The sums of numbers on each side of the magic triangle are all the same. Find two solutions for the magic triangle using a different number in each box.

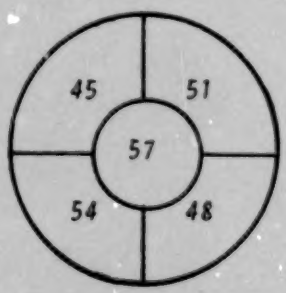


2. If 3600 boxes of kleenex are produced per hour, how many boxes are produced per second?

3. What is the sum of all the divisors of 5!?

4. a) How many shots at this target are needed to make a score of 300?

   b) Find four different combinations where the value of the shots on the target totals 300.



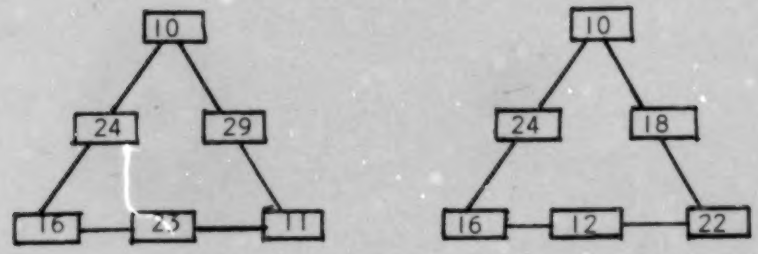
5. If Marie had three different skirts and four different sweaters, how many different outfits could she wear?

6. How many degrees does the minute hand of a clock pass through between 9:30 a.m. and 10:17 a.m.?

PROBLEM SHEET 7.2

ANSWERS AND SELECTED SOLUTIONS

1. Two possible solutions are:



2. One box per second.

3. $1 + 51 + 3 + 17 = 72$

4. a) 6

b) Four possible combinations are:

1)	2)	3)	4)
2 x 45	5 x 51	1 x 51	1 x 51
2 x 54	1 x 45	2 x 45	1 x 57
2 x 51		1 x 57	4 x 48
		1 x 48	
		1 x 54	

5. 12 outfits

6. $360 \div 60 \times 47 = 282°$

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PROBLEM SHEET 7.3

1. Using +, -, x, ÷, complete this number sentence:

   3 ☐ 4 ☐ 2 ☐ 1 = 7

2. If 17 x ☐ x 3 x 2 = 102, find ☐.

3. If the horizontal, vertical and diagonal sums are all the same, find the missing numbers.

24	3	18
	27	

4. Melanie received $5.50 for babysitting on Saturday. She gets $1.00 an hour and $1.50 per hour after midnight. How many hours did she babysit, if she went home at 1:00 a.m.?

5. Each letter represents a different prime number. Find each number. (There are three possibilities.)

   a + b - c = 41 where a, b, and c are each less than 30.

6. In a box there are twelve pieces of paper, each with a number. The first is numbered 1, the second 2, the third 3, and so on until 12. The box is shaken and the numbers drawn out in pairs. If the sums for each of the six pairs are 4, 6, 13, 14, 20, and 21, what are the numbers that make up the pairs?

PROBLEM SHEET 7.3
ANSWERS AND SELECTED SOLUTIONS

1. $3 \times 4 \div 2 + 1 = 7$.

2. $n = 1$

3.

24	3	18
9	15	21
12	27	6

4. 5 hours.

5. $19 + 29 - 7 = 41$
   $23 + 29 - 11 = 41$
   $17 + 29 - 5 = 41$

6.

Sum	Number Pair
4	1, 3
6	4, 2
13	6, 7
14	5, 9
20	8, 12
21	10, 11

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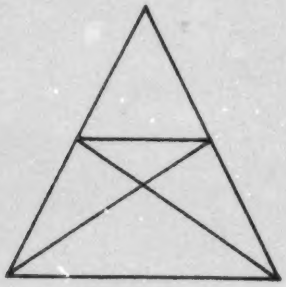
PROBLEM SHEET 7.4

1. Find the missing exponent.

$$5 \square \times 2^4 = 10\,000$$

2. The sum of two consecutive whole numbers is 37. What are the two numbers?

3. How many triangles are there in this figure?



4. If $a = 15$ and $a - b = 6$, find the value of $a + 2b$.

5. If r is less than 10, what value of r makes $r3749r0$ divisible by 60? (There are two possibilities.)

6. The sum of the ages of 2 children is 11 years. In 5 years one will be twice as old as the other. What are their ages now?

PROBLEM SHEET 7.4

ANSWERS AND SELECTED SOLUTIONS

1. 4

2. 18, 19

3. 12 triangles

4. $b = 9$, $a + 2b = 33$

5. $r = 2$ or $r = 8$

6. Try all possible ages whose sum is 11, then add five to each. The present ages are 2 and 9 years.

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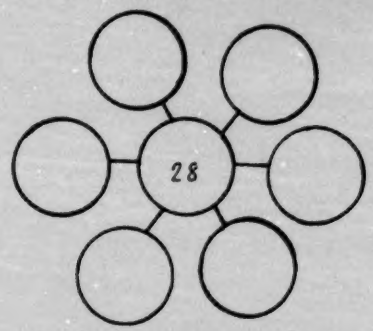
PROBLEM SHEET 7.5

1. Five, two and three are all factors of some numbers less than 100. Find three such numbers.

2. Which of the numbers in the following set divide evenly into 28?
{1, 2, 3, 4, 5, 8, 10}

3. Place the numbers 22, 24, 26, 30, 32, 34 in the circles so that the numbers of any three circles in a line have equal sums.



4. If $x = 5$ and $xy = 35$, find $x + y$.

5. Find the sum of the odd natural numbers listed. Look for a relationship between the number of odd natural numbers and their sum.

Odd Natural Numbers	Number of Natural Numbers	Sum
1	1	1
1, 3	2	4
1, 3, 5	3	
1, 3, 5, 7	4	
1, 3, 5, 7, 9		
1, 3, 5, 7, 9, 11		
1, 3, 5, 7, 9, 11, 13		
1, 3, 5, ..., 27, 29	15	
1, 3, 5, ..., 97, 99		

6. Ronald, who is collecting tropical fish, bought 13 fish for $16.00. He paid $4.00 each for some fighting fish, 50¢ each for small black mollies, and 25¢ each for some guppies. How many fish of each kind did Ronald buy?

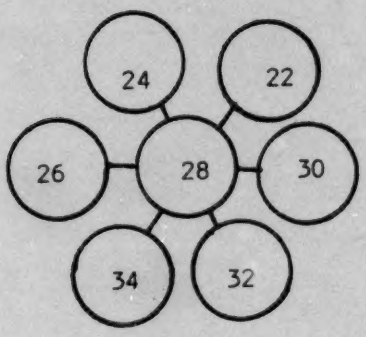
PROBLEM SHEET 7.5

ANSWERS AND SELECTED SOLUTIONS

1. 30, 60, 90

2. 1, 2, 4

3.



4. 12

5.

Odd Natural Numbers	Number of Natural Numbers	Sum
1	1	1
1, 3	2	4
1, 3, 5	3	9
1, 3, 5, 7	4	$4^2 = 16$
1, 3, 5, 7, 9	5	$5^2 = 25$
1, 3, 5, 7, 9, 11	6	$6^2 = 36$
1, 3, 5, 7, 9, 11, 13	7	$7^2 = 49$
1, 3, 5, ..., 27, 29	15	$15^2 = 225$
1, 3, 5, ..., 97, 99	50	$50^2 = 2500$

6. 3 fighting fish, 6 black mollies, 4 guppies

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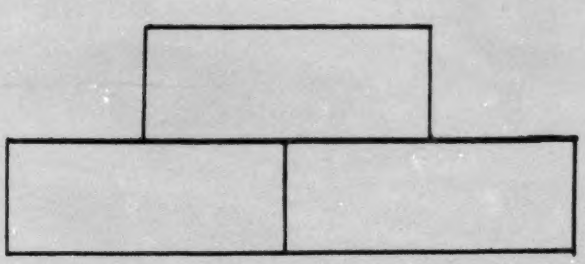
PROBLEM SHEET 7.6

1. Find a set of 3 consecutive even numbers whose sum is 294.

2. A clock strikes the time on the hour only. How many strikes will it make in one day?

3. Jean is thinking of a number. If she increases it by 24 and divides it by 3, the result will be the original number. What is the number that Jean is thinking of?

4. The three bricks in the diagram below are 6 cm wide and 20 cm long. Find the perimeter of the figure.



5. Steve, Michael, Sandra, and Lesley are standing in line to buy tickets for a movie. In how many ways can they stand in line to buy their tickets?

6. If $x^3 = 8$ and $y^3 = 64$, find the value of $x + y$.

PROBLEM SHEET 7.6

ANSWERS AND SELECTED SOLUTIONS

1. 96, 98, 100

2. $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12) \times 2 = 156$ strikes.

3. 12

4. $4 \times 6 + 4 \times 20 = 104$ cm

5. 24 ways

6. 6

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PROBLEM SHEET 7.7

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1. Find the missing number:

$$\frac{2 \times \square \times 28 \times 35}{49 \times 4} = 30$$

2. Find n if $n + n + n = 1$

3. Mrs. Pennel bought 3 dozen eggs. If her family eats $\frac{1}{4}$ of a dozen each day, how many days will the eggs last?

4. What is the smallest number which must be subtracted from 34 657 so that the difference is exactly divisible by 129?

5. Complete the sequence:

$$\frac{1}{4} \rightarrow \frac{7}{12} \rightarrow \frac{11}{12} \rightarrow 1\frac{1}{4} \rightarrow \square \rightarrow \square \rightarrow \square$$

6. The fencing that encloses a square lot is attached to upright posts placed 5 m apart. If twenty posts are necessary to fence the lot, what is its area?

PROBLEM SHEET 7.7

ANSWERS AND SELECTED SOLUTIONS

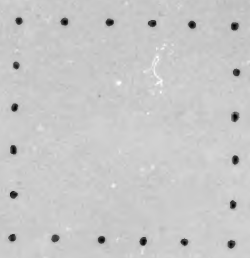
1. 3

2. $n = \frac{1}{3}$

3. 12 days

4. 85

5. $1\frac{7}{12}$, $1\frac{11}{12}$, $2\frac{1}{4}$

6. 

$25 \times 25 = 625\ m^2$

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PROBLEM SHEET 7.8

1. Find the values for A, B, and C in the following addition question.

$$\begin{array}{r} AA \\ +\ BB \\ \hline CBC \end{array}$$

2. Complete the square so that the rows, columns and diagonals all add to the same number.

1		$\frac{3}{4}$
	$\frac{5}{8}$	
		$\frac{1}{4}$

3. The sum of two numbers is 13 and their product is 42. Find the sum of the reciprocals of the two numbers.

4. If the perimeter of a square is 36 cm, find its area.

5. Two fathers and two sons divided a pie amongst them. Each received exactly $\frac{1}{3}$ of the pie. How was this possible?

6. In a hall, $\frac{1}{3}$ of the people present are men, $\frac{1}{4}$ of those present are women, and the rest are children. If there are 1152 people in the hall, how many children are there?

PROBLEM SHEET 7.8

ANSWERS AND SELECTED SOLUTIONS

1. $\begin{array}{r} 99 \\ +\ 22 \\ \hline 121 \end{array}$

2.

1	$\frac{1}{8}$	$\frac{3}{4}$
$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$
$\frac{1}{2}$	$1\frac{1}{8}$	$\frac{1}{4}$

3. $\frac{1}{6} + \frac{1}{7} = \frac{13}{42}$

4. Area = 81 cm^2

5. There was a grandfather, a father and a son.

6. Fraction of men and women is $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$.

   Fraction of children is $\frac{5}{12}$.

   There are 480 children.

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PROBLEM SHEET 7.9

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1. If $a = \frac{2}{5}$ and $b = 0.06$, find $a + b$.

2. Find the total amount of the following deposit: 9 quarters, 6 nickels, 13 pennies, 7 dimes, 4 one dollar bills, one five dollar bill, 6 ten dollar bills, and one twenty dollar bill.

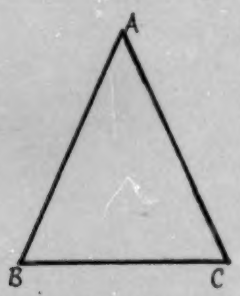
3. If you bought an article that cost \$5.13 and there was a charge of 25¢ sales tax on the cost, what would your change be if you paid with a ten dollar bill?

4. A boy attempts to climb a 10 m pole. At every attempt he climbs 1 metre and slips back $\frac{1}{2}$ metre. After how many attempts will he have reached the top?

5. Arrange the following numbers in order from smallest to largest.

$$\frac{1}{3}, \quad 0.34, \quad \frac{3}{10}, \quad 0.33$$

6. If AB = AC, BC = $3\frac{5}{8}$ m, and the perimeter of triangle ABC is $16\frac{1}{8}$ m, what are the lengths of the sides AB and AC?



PROBLEM SHEET 7.9

ANSWERS AND SELECTED SOLUTIONS

1. 0.46

2. $92.38

3. $4.62

4. 19 attempts

5. $\frac{3}{10}$, 0.33, $\frac{1}{3}$, 0.34

6. AB = AC = $6\frac{1}{4}$m

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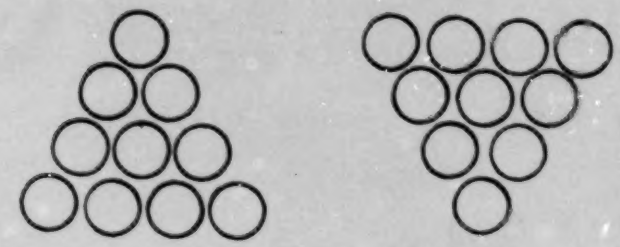
PROBLEM SHEET 7.10

1. The ratio of girls to boys in JoAnne's class is 3 to 4. If there are 12 girls, how many boys are there?

2. Ron worked for 48 hours at $14.50 an hour. Maureen worked for 40 hours at $15.00 an hour with time and half after 30 hours. Who earned more money? How much more?

3. Move three circles in the figure on the left to make it like the figure on the right.



4. Mr. Johnson's Honda uses 24 litres of gasoline to make a 288 km trip. How many litres of gasoline are needed to travel 432 km?

5. Of the three angles in a given triangle,
   a) how many right angles are possible?
   b) how many obtuse angles are possible?

6. A boy has a large bag of marbles. While he was counting them, he found he could divide them by 2, by 3, by 4 and by 6 and have one marble left over each time. However, when he divided them by 7 there were no marbles left over. How many marbles did the boy have if the bag cannot hold more than 100 marbles?

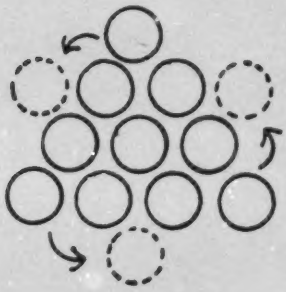
PROBLEM SHEET 7.10

ANSWERS AND SELECTED SOLUTIONS

1. 16 boys

2. Ron earned $21.00 more.

3.



4. 36 litres

5. a) 1 right angle
   b) 1 obtuse angle

6. 49 marbles

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PROBLEM SHEET 7.11

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1. Complete the sequence.

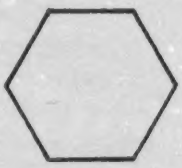
$0.03 \rightarrow \quad 0.06 \rightarrow \quad 0.09 \rightarrow \square \rightarrow \square \rightarrow \square \rightarrow \square$

2. Mr. Weiss, Mr. Baker, Mr. Perrault, Ms Shewchuk live in Ottawa, Toronto, Winnipeg and Vancouver but not necessarily in that order. Ms Shewchuk lives closer to Mr. Weiss than she does to the others. Mr. Weiss is a travel agent, Mr. Baker is a cable car operator, Mr. Perrault is an R.C.M.P. undercover agent and Ms Shewchuk works for the Governor General. In what city does each person live?

3. A baseball glove sells for $30.00. What would be the sale price if the store offered a discount of 20% on the regular price?

4. If a 10 kg bag of dog food costs $1.00 a kilogram and will feed one dog for 20 days, what will it cost to feed three dogs for 60 days?

5. Remove two sides from this regular hexagon. Then add three lines, each of the same length, to form two quadrilaterals.



6. A rectangular lot is worth $10 000.00. What is the value of a similar lot whose length and width are 50% greater?

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PROBLEM SHEET 7.11

ANSWERS AND SELECTED SOLUTIONS

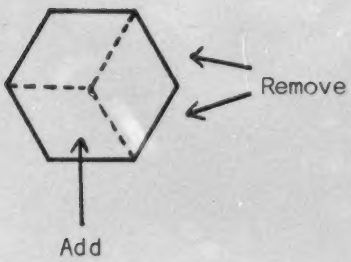
1. 0.12, 0.15, 0.18, 0.21

2. Mr. Weiss lives in Toronto.
   Ms Shewchuk lives in Ottawa.
   Mr. Baker lives in Vancouver.
   Mr. Perrault lives in Winnipeg.

3. $24.00

4. $90.00

5.



6. If original area is (1) l x (1) w = (1) A,
   then 50% increase in l and w gives

$$(\tfrac{3}{2})l \times (\tfrac{3}{2})w = (\tfrac{9}{4})A$$

Area increase is $\frac{9}{4}$ or $2\frac{1}{4}$ times.

Value of lot is 10 000 x $\frac{9}{4}$ = $22 500.00

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PROBLEM SHEET 7.12

1. Two numbers are in the ratio 2:5. Their sum is 42. Find the numbers.

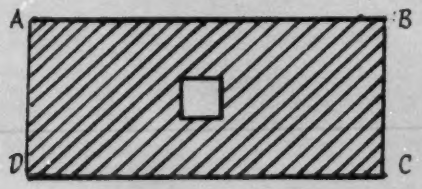
2. The numbers 2, 4, 6, 8 are placed in the empty squares so that each row, each column, and each diagonal contains each of the four numbers. Find the sum of A and B. There are two possibilities.

2	4	6	8
A	B		

3. Find the number halfway between $1\frac{3}{4}$ and $3\frac{5}{8}$.

4. In a class of 25 students, 20% wear glasses. Two of those wearing glasses are left-handed. Of those wearing glasses what per cent are left-handed?

5. Rectangle ABCD has length 20 cm and width 10 cm. Inside the rectangle is a square with side 3 cm. Find the area of the shaded region.



6. The cost of a radio is $30.03 including 5% sales tax. What was the original price of the radio?

PROBLEM SHEET 7.12

ANSWERS AND SELECTED SOLUTIONS

1. 12, 30

2.

2	4	6	8
8	6	4	2
4	2	8	6
A	B	2	4

$A + B = 6 + 8$

$= 14$

2	4	6	8
6	8	2	4
8	6	4	2
A	B	8	6

$A + B = 4 + 2$

$= 6$

3. $2\frac{11}{16}$

4. 40%

5. 191 cm^2

6. Original price + sales tax = total price

$100 + 5 = 105\%$

105% of original price = 30.03

1.05 x original price = 30.03

Original price is $28.60.

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CALCULATOR PROBLEM SHEET 7.13

1. Find the missing numbers.

$$\begin{array}{r} \square\,3\,2\,4 \\ \times\ 4\,\square \\ \hline 2\,5\,5\,5\,5\,2 \end{array}$$

2. Mr. Simon bought a calculator priced at $28.98. If the provincial sales tax is 5%, how much change would he receive from a fifty dollar bill?

3. A rectangle and a square have the same area. If the dimensions of the rectangle are 10.4 cm by 2.6 cm, find the dimensions of the square.

4. Find the missing numbers.

a) $\dfrac{10^{7}}{10^{\square}} = 100$ b) $\dfrac{10^{6}}{5^{2.\square}} = 10\,000$

5. Find a number that, when multiplied by itself, falls between 2.99 and 3.01.

6. Four classes went on a field trip to Birds Hill Park. Three quarters of one class went horseback riding; 67% of the second class went on the cedar bog trail; 0.68 of the third class went swimming. If the average participation of the four classes in activities is 65%, find what portion of the fourth class was responsible for setting up the barbecue.

CALCULATOR PROBLEM SHEET 7.13

ANSWERS AND SELECTED SOLUTIONS

1.
$$\begin{array}{r} \boxed{5}\,3\,2\,4 \\ \times\quad 4\,\boxed{8} \\ \hline 2\,5\,5\,5\,5\,2 \end{array}$$

2. $50.00 - (28.98 + 0.05 \times 28.98) = 50.00 - 30.43$
$= \$19.57$

Change is \$19.57.

3. Side of square $= \sqrt{(10.4)(2.6)}$
$= 5.2$ cm

4. a) 5 b) 2^2 or 4

5. There is an infinite number of solutions. For example, 1.73 and $\sqrt{3} \doteq 1.7320508$.

6. $4 \times 0.65 = 2.6$
$2.6 - (0.75 + 0.67 + 0.68) = 0.5$
Portion of fourth class is 0.5.

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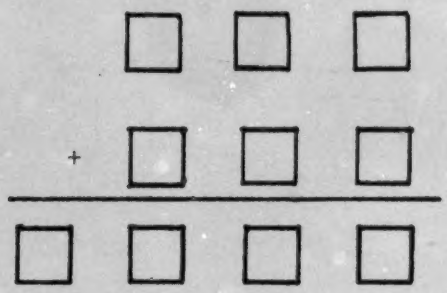
PROBLEM SHEET 7.14 - SUPERSTAR

1. Use each of ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, exactly once in the problem to make the sum correct.

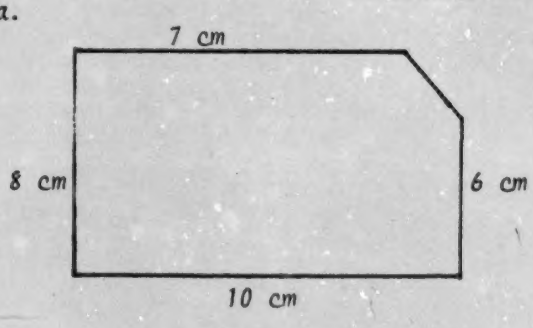


2. Rearranging the digits of the number 734 produces many different numbers. Find the sum of all such numbers.

3. Show how to plant 10 trees so that there are five lines of four trees each.

4. A total of 28 handshakes were exchanged at a party. Each person shook hands exactly once with each of the others. How many people were present at the party?

5. Find the area.



6. On planet CRYPTON, ten per cent of the population have two computers each, half of the remainder have none, and all the others have one computer each. How many computers are there on planet CRYPTON if there are 40 000 inhabitants?

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PROBLEM SHEET 7.14 - SUPERSTAR

ANSWERS AND SELECTED SOLUTIONS

1. One possible solution is:

	7	8	9
+	2	6	4
1	0	5	3

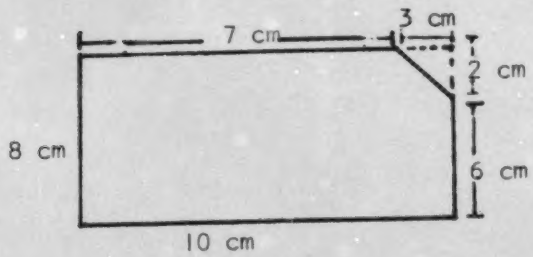
2. 3108

3.

4. 8 people

2 people yields 1 handshake
3 people yields 3 handshakes
4 people yields 6 handshakes
5 people yields 10 handshakes
8 people yields 28 handshakes

5.



$$\text{Area} = (10)(8) - \frac{1}{2}(2)(3)$$
$$= 80 - 3$$
$$= 77 \text{ cm}^2$$

6. Number of inhabitants with two computers is: 10% of 40 000 = 4000

Number of computers is: 4000 x 2 = 8000

Number of inhabitants with one computer is: 36 000 x $\frac{1}{2}$ = 18 000

Number of computers is: 18 000 x 1 = 18 000

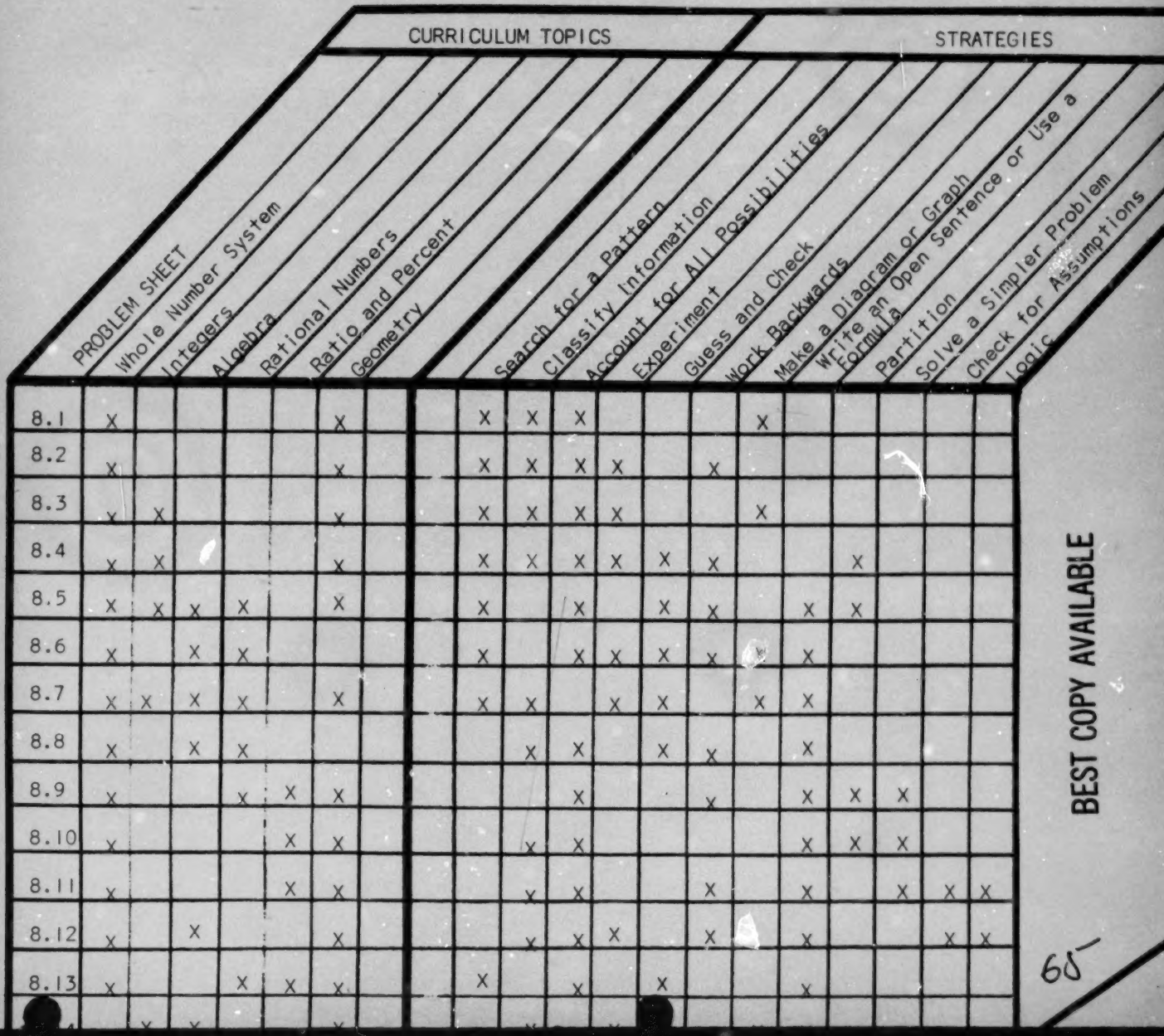
Total number of computers is: 8000 + 18 000 = 26 000

grade 8

problem sheets

(with answers and selected solutions)

CURRICULUM CHART: GRADE 8


CURRICULUM TOPICS
STRATEGIES
PROBLEM SHEET
Whole Number System
Integers
Algebra
Rational Numbers
Ratio and Percent
Geometry
Search for a Pattern
Classify Information
Account for All Possibilities
Experiment
Guess and Check
Work Backwards
Make a Diagram or Graph
Write an Open Sentence or Use a Formula
Partition
Solve a Simpler Problem
Check for Assumptions
Logic
8.1 X X X

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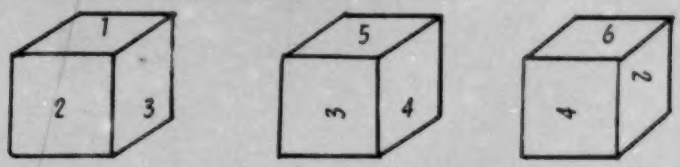
PROBLEM SHEET 8.1

1. What is the sum of all the divisors of 12?

2. State the next three numbers in the sequence 1, 6, 16, 31, ...

3. Find the sum: 1 + 3 + 5 + ... + 95 + 97 + 99.

4.

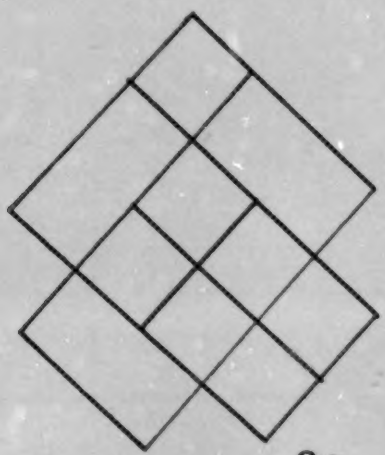


These are three views of the same cube.

a) Which side is opposite side 4?
b) Which side is opposite side 5?
c) Which side is opposite side 6?

5. At a party, three 8-slice pizzas are ordered by five students. If each student eats the same number of slices, what is the minimum number of slices that would be left over?

6. How many squares are there in this figure?



PROBLEM SHEET 8.1

ANSWERS AND SELECTED SOLUTIONS

1. $1 + 2 + 3 + 4 + 6 + 12 = 28$

2. 51, 76, 106

3. 2500

4. a) 1
   b) 2
   c) 3

5. 4 slices

6. 13 squares

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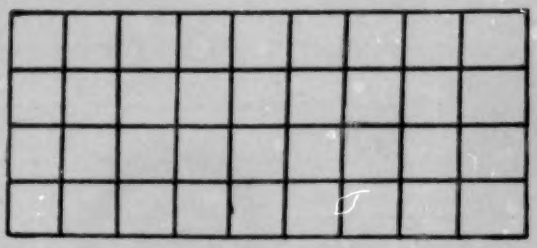
PROBLEM SHEET 8.2

1. Find the largest whole number less than 1382 that is divisible by 9.

2. A woman bought some 40 cent stamps, ten times as many 20 cent stamps, and some dollar stamps that cost $20.00 altogether. How many of each kind of stamp did the woman buy?

3. Using a piece of graph paper, show how to cut the rectangle into two pieces that will fit together to form a square.



4. Give the next three numbers in each of the following series:
   a) 1, 4, 9, 16, ___, ___, ___.
   b) 1, 8, 27, 64, ___, ___, ___.
   c) 0, 7, 26, 63, ___, ___, ___.

5. What is the last digit in 3^{25}?

6. Art, Brian, Carl, and Don each went on a total of four dates with four different girls. The second time, Art dated Effie and Carl dated Froncine. The third time, Brian dated Hattie and Don dated Georgia. The fourth time Carl dated Georgia and Don dated Francine. For each of the four times, tell which girl dated which boy.
   HINT: Use the chart to solve the problem.

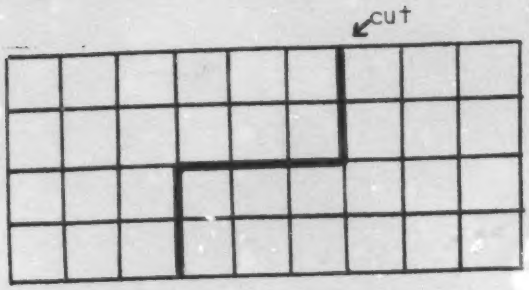
	1	2	3	4
Art				
Brian				
Carl				
Don				

PROBLEM SHEET 8.2

ANSWERS AND SELECTED SOLUTIONS

1. 1377

2. 5 forty cent stamps; 50 twenty cent stamps; 8 dollar stamps.

3. 

4. a) 25, 36, 49
   b) 125, 216, 343
   c) 124, 215, 342

5. $3^1 = 3$ $3^5 = 243$ $3^{25} = \ldots3$
   $3^2 = 9$ $3^6 = \ldots.9$
   $3^3 = 27$ $3^7 = \ldots.7$
   $3^4 = 81$ $3^8 = \ldots.1$

6.

	1	2	3	4
Art	Georgia	Effie	Francine	Hattie
Brian	Francine	Georgia	Hattie	Effie
Carl	Hattie	Francine	Effie	Georgia
Don	Effie	Hattie	Georgia	Francine

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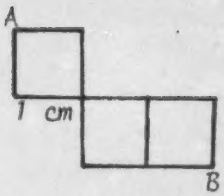
PROBLEM SHEET 8.3

1. State the value of:

   a) 100 - 99 + 98 - 97 + 96 - 95 ..... + 4 - 3 + 2 - 1

   b) 100 + 99 + 98 + 97 + 96 + 95 ..... + 4 + 3 + 2 + 1

2.



How many different 5 cm paths are there from point A to point B?

3. How many four digit licence plates can you make with the sum of these four digits equal to 35? A digit may be used more than once in the same licence plate.

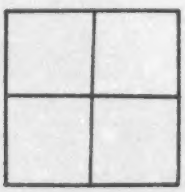
4. Twelve toothpicks are arranged as in the diagram below. Can you make the following arrangements?

   a) Remove four toothpicks to form two squares.

   b) Remove four toothpicks to form one square.

   c) Remove two toothpicks to form two squares.

   d) Move three toothpicks to form three squares.



5. There is a set of 1961 different integers. The sum of all these integers is 1962. At most, how many of the integers in this set can be odd?

6. If a student were to calculate the product of all natural numbers from 1 to 25 inclusive (i.e., 1 x 2 x 3 x ... x 24 x 25), how many zeros would there be at the end of this product?

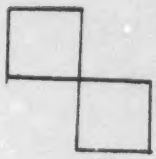
PROBLEM SHEET 8.3

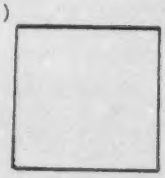
ANSWERS AND SELECTED SOLUTIONS

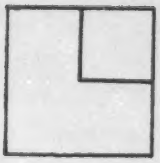
1. a) 50
   b) 5050

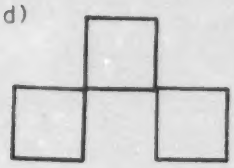
2. 6 paths

3. 4 (The possible combinations are 9998, 8999, 9899, 9989.)

4. a) 

   b) 

   c) 

   d) 

5. The sum of any two odd numbers is even. Therefore, the maximum number of odd numbers is 1960.

6. For a number to end in zero it must have a factor of 10 or factors of 2 and 5. Here there are several factors of 2, but only 6 factors of 5. Therefore, there are 6 zeros at the end of the product.

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PROBLEM SHEET 8.4

1. John observed the temperature to be -2°C at 9:00 a.m. A storm was approaching Portage la Prairie and the temperature dropped 3°C per hour until 11:00 a.m. Just before noon the temperature rose 5°C. What is the noon temperature?

2. Note that $1 + 3 = 4$

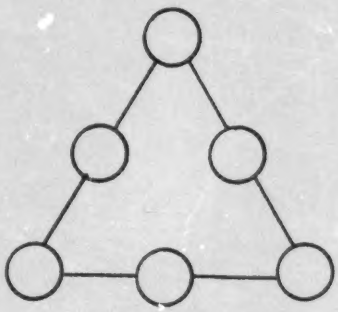
   $1 + 3 + 5 = 9$

   $1 + 3 + 5 + 7 = 16$

   a) What is the sum of the first 20 odd whole numbers?

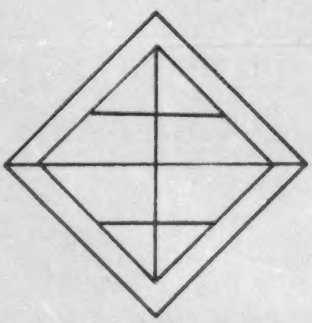
   b) What is the sum of the first 100 odd whole numbers?

3. Draw the figure. Arrange the numbers 4, 5, 6, 40, 50, and 60 in the circles in such a way that the product of the three numbers on each side of the triangle is 1200.



4. A person has a dollar bill and wants change. He receives 50 coins that total exactly $1.00. How many of each coin did he receive? (There are two possibilities.)

5. How many different triangles can be found in this figure?



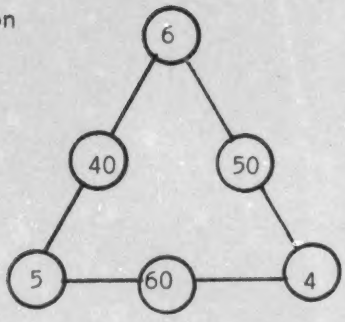
6. Bill and Jane work evenings. Bill is off duty every ninth evening; Jane is off every sixth. Bill is off tonight and Jane is off tomorrow. When will they be off duty on the same evening?

PROBLEM SHEET 8.4
ANSWERS AND SELECTED SOLUTIONS

1. -3°C

2. a) 400
   b) 10 000

3. One possible solution



4. i) 45 pennies, 2 nickels, 2 dimes, 1 quarter
   ii) 40 pennies, 8 nickels, 2 dimes

5. 16 triangles

6. They will never be off duty on the same evening as illustrated in the following table representing the pattern of days off:

B	J						J		B		
1	2	3	4	5	6	7	8	9	10	11	12
	↓					B	↓				
13	14	15	16	17	18	19	20	21	22	23	24
	↓		B				↓				
25	26	27	28	29	30	31	32	33	34	35	36
	↓						↓		B		
37	38	39	40	41	42	43	44	45	46	47	48
	↓										
49	50	. . .									

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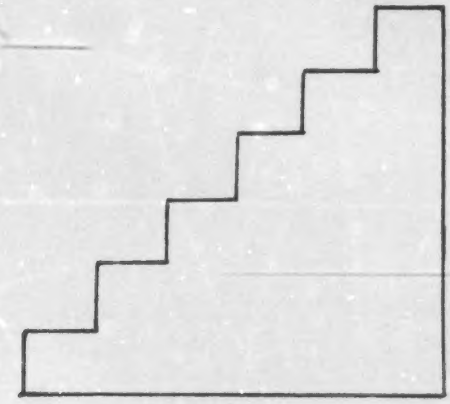
PROBLEM SHEET 8.5

1. Potato chips cost \$0.25 a bag and a coke costs \$0.40. How many of each can be purchased if exactly \$2.20 is spent?

2. Find the numbers in the next row of this array:

3. There are three consecutive odd integers. Four times the smallest is equal to the sum of the other two. Find the smallest of the three integers.

4. Find the area of the figure below. The line segments for the "step" formation meet at right angles and are of 1 cm length.



5. The three-digit number 2a3 is added to the number 326 to give the three-digit number 5b9. If 5b9 is divisible by 9, find a + b. (Both a and b are positive integers.)

6. If $a + b = 3$ and $a^2 + b^2 = 65$, find ab.

PROBLEM SHEET 8.5

ANSWERS AND SELECTED SOLUTIONS

1. 4 bags of potato chips and 3 cokes

2. 1 6 15 20 15 6 1

3. 3

4. 21 cm^2

5.
$$\begin{array}{r} 326 \\ +\ 2a3 \\ \hline 5b9 \end{array}$$

Since 5b9 is divisible by 9, b = 4

Since 2 + a = 4, a = 2

Therefore a + b = 2 + 4 = 6

6. Using guess and check, a = 4 and b = -7 or a = -7 and b = 4. Therefore, ab = -28.

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PROBLEM SHEET 8.6

1. Nine coins have a value of $1.35. If the coins are quarters and dimes, find the number of dimes.

2. Don, Ken, JoAnne, Marlene, and Peter wrote a mathematics test. Their average mark was 71%. Don, Ken, JoAnne, and Marlene received 74%, 80%, 95%, and 63% respectively. Peter didn't want to tell his mark. What was his mark?

3. Will two 5 cm drains give better, worse or the same drainage as a 10 cm drain? (The formula for the area of a circle is not required for the solution.)

4. In the following sequence, find the next three numbers.

$$\frac{2}{5},\ \frac{7}{12},\ \frac{19}{31},\ \frac{50}{81},\ \_\_,\ \_\_,\ \_\_.$$

5. The seats in my mathematics class are arranged in rows with the same number of seats in each row. Every seat is occupied. I am seated fourth from the front and second from the back. I am the fifth student from the left side of the room and the third student from the right side of the room. How many students are in my classroom?

6. If $a*b = 2a + b$, find $3*4$.

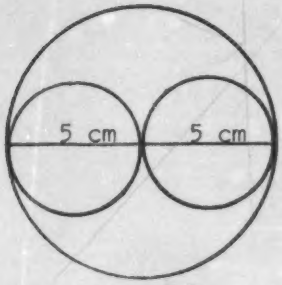
PROBLEM SHEET 8.6

ANSWERS AND SELECTED SOLUTIONS

1. 6 dimes

2. 43%

3.



The two 5 cm drains will give worse drainage than a 10 cm drain.

4. $\frac{50 + 81}{81 + 131}$, $\frac{131 + 212}{212 + 343}$, $\frac{343 + 555}{555 + 898}$,

$\frac{131}{212}$, $\frac{343}{555}$, $\frac{898}{1453}$,

5. 35 students

6. $a*b = 2a + b$

$3*4 = 2(3) + 4$

$= 10$

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PROBLEM SHEET 8.7

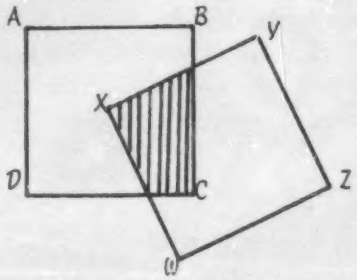
1. A student goes to a store with $40.00 to buy 50 items. If 1 pizza costs $5.00, a carton of coke costs $3.00 and 1 bubble gum costs $0.10, how many of each item can the student buy?

2. Find a rational number equal to $\frac{3}{11}$ such that the sum of the denominator and numerator is 84.

3. Multiply: $(1 - \frac{1}{3})\ (1 - \frac{1}{4})\ (1 - \frac{1}{5})\ (1 - \frac{1}{6})$

4. A mini-bike can go 38 km on one litre of gasoline. Its tank holds 5.8 litres. How many kilometres can the bike go on 0.6 of a tank of gasoline?

5. Square ABCD and square WXYZ each have a side of 10 cm. X is the centre of square ABCD. Find the area of the shaded region.



6. Write an expression using three -9's to equal +10.

PROBLEM SHEET 8.7

ANSWERS AND SELECTED SOLUTIONS

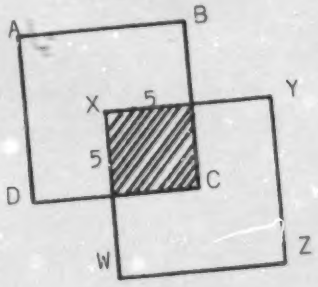
1. 3 pizzas, 7 cartons of coke, 40 bubble gum

2. $\frac{18}{66}$

3. $\frac{1}{3}$

4. 132.24 km

5. The area of the shaded region remains constant as the square WXYZ is rotated about X.



Area of shaded region is 25 cm^2.

6. $\frac{-9}{-9}$ - (-9)

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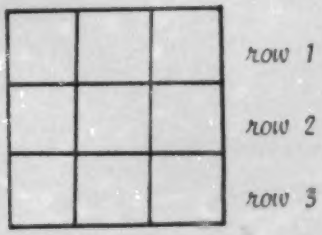
PROBLEM SHEET 8.8

1. Find the value of "k".

$$(\tfrac{1}{2} \times 60) + (\tfrac{1}{2} \times 8) = \tfrac{1}{2} k$$

2. If each of the numbers 3, 4, 5, 6, 7, 8, 9, 10, 11 are placed in separate squares in the diagram so that the sums of the rows are equal, what will be the sum of each row?



3. Jane spent $\frac{1}{2}$ of her money on a junior high dance ticket. One half of the remaining money was used for bus fare. She arrived home with \$0.25. What amount of money did she have originally?

4. Rearranging the digits of the number 579 produces different numbers. Find the sum of all such numbers, including 579.

5. By what number must 12 be multiplied to make it the smallest:
   a) perfect square?
   b) perfect cube?

6. Three friends ate lunch together. One spent \$3.60 for a glass of milk, two hamburgers, and a doughnut. The second bought two glasses of milk, one hamburger, and two doughnuts for \$3.00. If the third bought one glass of milk, one hamburger and one doughnut, how much did he pay?

PROBLEM SHEET 8.8

ANSWERS AND SELECTED SOLUTIONS

1. 68

2. The solution may be obtained by placing the numbers in the diagram or by dividing the sum of the numbers by 3. The sum of each row is 21.

3. $1.00

4. 4662

5. a) 3
   b) 18

6. 3 milk + 3 doughnuts + 3 hamburgers = $6.60

   Therefore, 1 milk + 1 doughnut + 1 hamburger = $\frac{\$6.60}{3}$

   = $2.20.

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PROBLEM SHEET 8.9

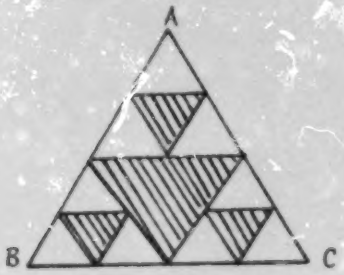
1. The average speed of the cars during a race at the Winnipeg Speedway is 162 km/h. Adding the average speed of each driver that entered the race results in the sum of 1782 km/h. If the race is 30 laps in length, how many drivers entered the race?

2. In a class of 28 students, the ratio of boys to girls is 3 to 4. How many girls are there in the class?

3. Assume four out of five dentists recommend sugarless gum. If 200 dentists were surveyed, how many would recommend sugarless gum?

4. Eager Eagore tosses three coins: a quarter, a dime, and a nickel. The result of his first round of tosses is recorded as follows: quarter-heads, dime-tails, nickel-tails. State the total number of possibilities when the 3 coins are tossed.

5. In the diagram, triangles of all sizes are equilateral.
   a) If the area of $\triangle ABC$ is 16 cm^2, find the total area of the shaded regions.
   b) What fraction of $\triangle ABC$ is unshaded?



6. In a class of 30 students, $\frac{1}{3}$ received a mark of 60% or less on a mathematics test, while $\frac{3}{5}$ obtained a mark of between 60% and 90%. How many students achieved a mark of 90% or more?

PROBLEM SHEET 8.9

ANSWERS AND SELECTED SOLUTIONS

1. $1782 \div 162 = 11$ drivers

2. 16 girls

3. 160 dentists

4. 8 possibilities

5. a) $7\ cm^2$

   b) $\frac{9}{16}$

6. 2 students

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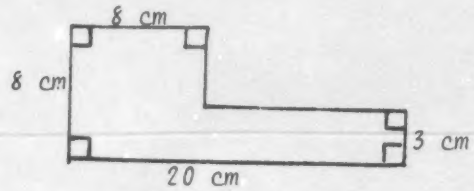
PROBLEM SHEET 8.10

1. One out of every three seats on an airplane is empty. There are 68 passengers on the plane. How many empty seats are there on the plane?

2. A class of 20 students averages 66% on an examination; another class of 30 students averaged 56%. Find the average for all students.

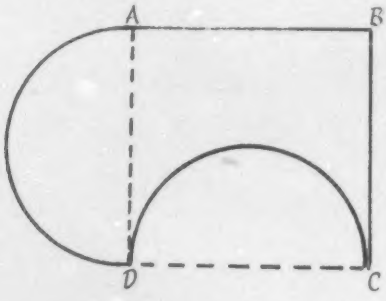
3. Find the area of the following figure:



4. In a barn there are some hens and pigs. They have 13 heads and 34 legs altogether. How many hens and how many pigs are there in the barn?

5. If you were given one million dollars and told to spend the money at the rate of one dollar per minute, how long would it take you to spend all of the money? Give the answer to the nearest day.

6. In the accompanying figure, $\angle B$ is a right angle and $AD = BC = AB = DC = 14$ cm. AD and DC are diameters of the semicircles. Find the area of the figure formed by the solid lines ($\pi = 3.14$).



PROBLEM SHEET 8.10

ANSWERS AND SELECTED SOLUTIONS

1. Empty seats:full seats := 1 : 2

   n:68 = 1 : 2

   n = 34

2. 60%

3. 100 cm^2

4. 9 hens, 4 pigs

5. 695 days

6. 196 cm^2

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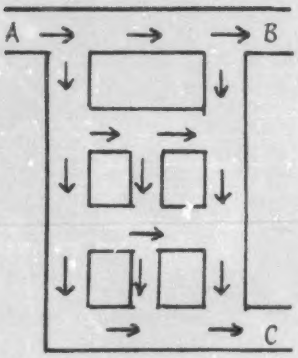
PROBLEM SHEET 8.11

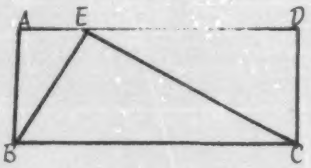
1. The diagram shows a network of one-way streets. At each intersection, the traffic divides equally between the two directions. All cars exit through B or C. If 304 cars enter the network at point A, how many of these cars exit from C?



2. If a boy can mow a lawn 100 m x 100 m in 60 minutes, how long will it take him to mow a lawn 50 m x 50 m?

3. One hundred fifty-six pennies are to be divided among three boys in the ratio of 6 to 4 to 3. How many pennies will each boy receive?

4. In the accompanying diagram the area of rectangle ABCD is 15 cm^2. If AD = 5 cm, find the area of triangle BEC.



5. There are nine people in a room. Each one shakes hands with each of the others once. How many handshakes are there?

6. In an economy move at a factory, all the workers had their salaries cut by 10%. They were all very unhappy about this and threatened to go on strike. At this point, the factory president stepped in and agreed to a 10% raise on their new salary. "Everyone should be happy again," he said with a smile. "Things are back the way they were to start with." Were they? Why?

PROBLEM SHEET 8.11

ANSWERS AND SELECTED SOLUTIONS

1. 228 cars

2. 15 minutes

3. 72, 48, 36

4. 7.5 cm^2

5. 36 handshakes

6. No. Assume a salary base of \$100.00.
   A 10% reduction reduces the salary to \$90.00.
   A 10% increase on \$90.00 is \$9.00, resulting in a salary of \$99.00.
   The loss is 1% or \$1.00 on a base salary of \$100.00.

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PROBLEM SHEET 8.12

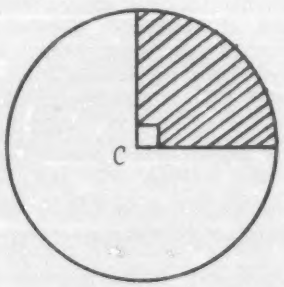
1. Find the maximum number of pieces when 5 cuts are made all the way across a circular cake, without cutting through the same point more than twice.

2. Ms Smith paid $8.40 for a book on sale. If she received a 30% discount, what was the original price of the book?

3. What are the dimensions of a quadrilateral which will enclose the largest area with 40 cm of string?

4. Ten books of 100 pages each are arranged in order on a shelf. A bookworm starts on page 1 of the first book and eats through the 100th page of the last book. How many pages has it eaten through? (Exclude covers and assume that one page equals one sheet.)

5. Find the area of the shaded region if C is the centre of the circle with radius 10 cm. (Use $\pi = 3.14$.)



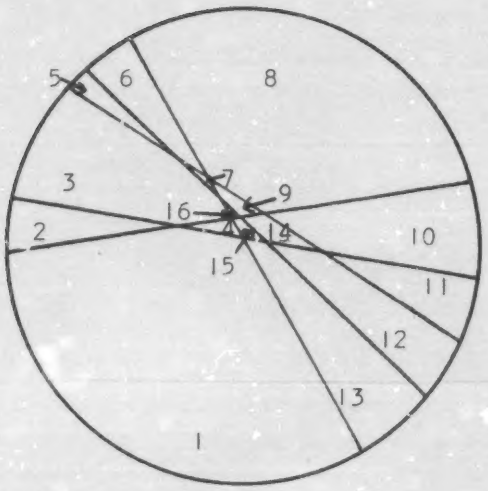
6. When Linda, Ron and Bill eat out, each orders either a milkshake or coke.
   a) If Linda orders a milkshake, Ron orders a coke.
   b) Either Linda or Bill orders a milkshake but not both.
   c) Ron and Bill do not both order coke.
   d) Linda and Bill do not both order coke.

   Who could have ordered a milkshake yesterday and a coke today? Explain your reasoning.

PROBLEM SHEET 8.12

ANSWERS AND SELECTED SOLUTIONS

1. 16 pieces



2. $12.00

3. 10 cm x 10 cm

4. Use books to solve the problem. The worm eats through 802 pages.

5. 78.5 cm^2

6.

	Yesterday	Today
Linda	C	C
Bill	M	M
Ron	M	C

Therefore, Ron ordered a milkshake yesterday and a coke today.

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CALCULATOR PROBLEM SHEET 8.13

1. a) Find the quotient when 2 is divided by 9, 99, and 999.
   b) Use the pattern established in (a) to predict what will happen when 2 is divided by 999 999 999.

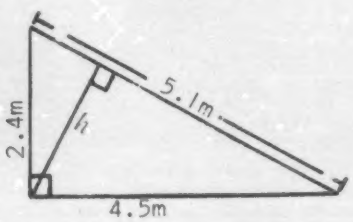
2. Find N as a decimal fraction that satisfies $2.59 < N^2 < 2.61$.

3. What two natural numbers less than 15 have a quotient of 0.272727...?

4. Write 8 406 398 as a product of prime factors.

5. Karen received $12.50 an hour plus commission. Her sales for the week were $2500.00. If her total earnings for a 40 hour week were $631.25, what is her rate of commission?

6. Find h to the nearest tenth.



CALCULATOR PROBLEM SHEET 8.13

ANSWERS AND SELECTED SOLUTIONS

1. a) $0.\overline{2}$; $0.\overline{02}$; $0.\overline{002}$.

   b) $0.\overline{000000002}$

2. There are several answers. One possible solution is 1.61.

3. 3 and 11

4. $2 \times 7 \times 11 \times 13^2 \times 17 \times 19$

5. Salary for 40 hour week is $40 \times 12.50 = \$500.00$.

   Amount of commission is $631.25 - 500.00 = \$131.25$.

   Rate of commission is $\frac{131.25}{2500.00} = 5.25\%$.

6. $\frac{1}{2}(2.4)(4.5) = \frac{1}{2}(5.1)(h)$

   $h = 2.1$ m (nearest tenth)

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PROBLEM SHEET 8.14 - SUPERSTAR

1. 2^n divides evenly into 1440, but 2^{n+1} does not. What is the value of n?

2. What is the base of the system in which the number 25 is doubled by reversing its digits?

3. A cow is tethered by a piece of rope 50 m long. The rope is fastened to a hook which is located 10 m from the corner on the longer side of the barn, which measures 60 m by 30 m. Over how much ground can the cow graze? (Use $\pi = 3.14$)

4. Let k be the number of positive integers that leave a remainder of 29 when divided into 204. Find k.

5. In order to decide which of three men should be his new premier, a king told the three that while they were blindfolded, he would put on each of their foreheads either a blue or a red star. He further told them that any one who saw one red star when the blindfold was removed was to raise his hand, and the first one to decide the color on his own forehead was to take his hand down. The first one with his hand down, who could also offer a correct explanation of his action, would be the new premier.

   The king put a red star on each of their foreheads. When the blindfolds were removed all three put their hands up, and then one took his hand down. How did the man taking his hand down decide on the color he had on his forehead?

6. Eighteen bottles are placed in a 4 x 6 drink case. Every row and column contains an even number of bottles. Draw a diagram showing the arrangements of drink bottles in the case.

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PROBLEM SHEET 8.14 - SUPERSTAR

ANSWERS AND SELECTED SOLUTIONS

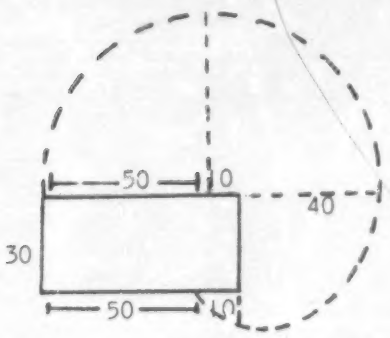
1. Express 1440 as a product of prime numbers, (2^5) ( ).(5). Then 2^5 is the largest power of 2 that divides evenly into 1440. Hence $2^n = 2^5$ and $n = 5$.

2.

Base (n)	Value of 25_n in base 10	Value of 52_n in base 10
6	17	32
7	19	37
8	21	42

Since 42 is twice 21, then 52_8 is twice 25_8.

3.



Grazing area $= \frac{1}{2}\pi(50)^2 + \frac{1}{4}\pi(40)^2 + \frac{1}{4}\pi($

$= \frac{1}{2}(3.14)(2500) + \frac{1}{4}(3.14)(16$

$\frac{1}{4}(3.14)(100)$

$= 3925 + 1256 + 78.5$

$= 5259.5\ m^2$

4. $204 - 29 = 175$

Express 175 as a product of prime numbers, (5) (5) (7).
Hence, $204 = (5)(5)(7) + 29$
Then 5, 7, 25, 35, and 175 divide into 204 and leave a remainder.
However, only 35 and 175 leave a remainder of 29 when divided into 204.
Therefore $k = 2$.

5. Suppose A and B had red and C had blue. In this case both A and B would immediately be aware that the other person had red since both would notice that C had blue. Since this did not occur and neither A nor B dropped their hands, C realized that all of them must have red, including himself.

6. One possible solution is:

(HINT: determine the 6 empty positions first.)

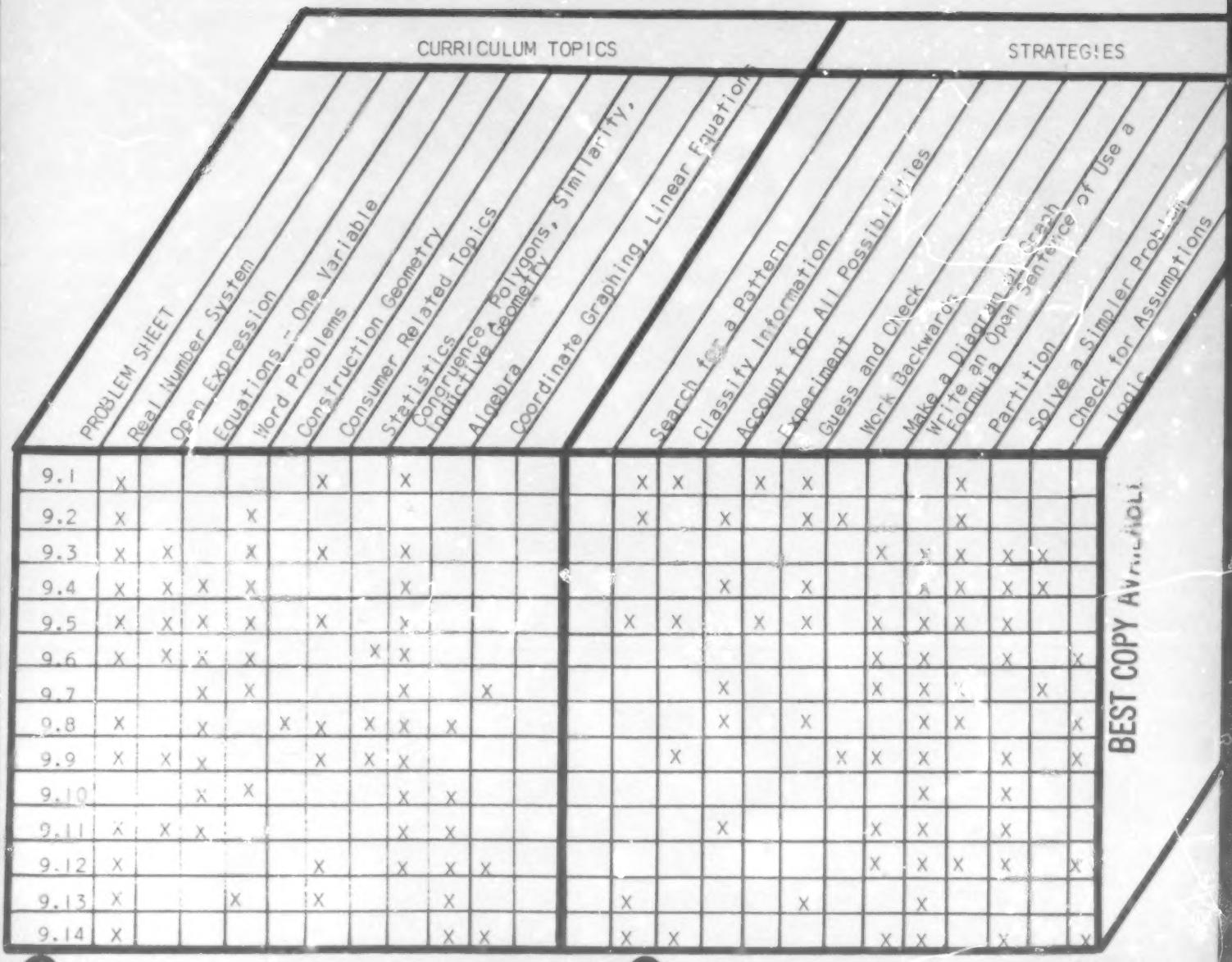
x	x	x	x
	x		x
		x	x
x			x
x	x	x	x
x	x	x	x

grade 9

problem sheets

(with answers and selected solutions)

CURRICULUM CHART: GRADE 9



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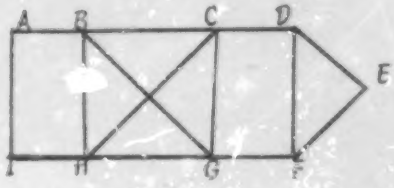
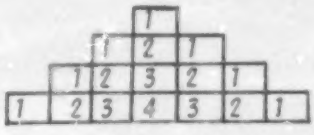
PROBLEM SHEET 9.1

1. Joe went shopping with $10.00 in his pocket. He spent half of his money in the first store and $1.70 in a second. He spent one third of the rest of his money in a third store. How much money has he left?

2. Draw the following figure without lifting your pen from the paper and without going over the same path twice. Where would the starting and stopping points be?



3.



Consider a larger tower with 1000 rows. The bricks in this tower are numbered in the same manner as those above.

a) Extend the "tower" by at least 2 more rows.

b) What is the number on the middle brick of row 3, row 4 and row 99?

4. Refer to the tower in no. 3 above.

a) The sum of the numbers on the bricks in row 2 is 4 (1 + 2 + 1 = 4). Find the sum for rows 3, 4, 5, and 6. Record your answers in the table.

Row number	1	2	3	4	5	6
Sum	1	4				

b) Now predict the sum for row 7.

c) What is the sum of the numbers on the bricks in row 900?

5. Refer to the tower in no. 3 above.

a) Notice there is one brick in row 1 and three bricks in row 2. Find the number of bricks in rows 3, 4, 5, and 6. Record your answers in the table.

Row number	1	2	3	4	5	6
Number of bricks	1	3				

b) Now predict the number of bricks in row 7

c) How many bricks would you expect in row 1000?

d) How many bricks have the number 100 on them if there are 1000 rows?

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PROBLEM SHEET 9.1

ANSWERS AND SELECTED SOLUTIONS

1. $2.20

2. There are two possibilities.

   Begin at D and end at F or vice versa since D and F are the only vertices which have an odd number of lines (3) emerging from them.

3. a)

	1	2	3	4	5	4	3	2	1	
1	2	3	4	5	6	5	4	3	2	1

   b) 3, 4, 99

4. a)

Row Number	1	2	3	4	5	6
Sum	1	4	9	16	25	36

   b) 49

   c) 900 x 900 = 810 000

5. a)

Row Numbers	1	2	3	4	5	6
Number of Bricks	1	3	5	7	9	11

   b) 13

   c) (1000 x 2)- 1 = 1999

   d)

Row number	Number of Hundreds
0 to 99	0
100	1
101 to 1000	900 x 2 = 1800

   Total number of hundreds is 1801.

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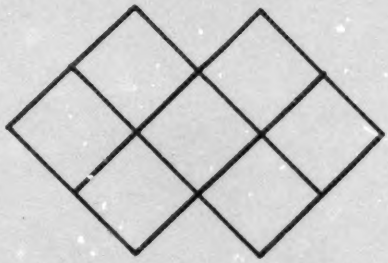
PROBLEM SHEET 9.2

1. Put the numbers 1 to 7 in the seven boxes so that the numbers in each line of three boxes add up to 10. Each number can be used only once.



2. Express $\frac{1}{81}$ as a repeating decimal.

3. If n is a positive integer less than 10, what value of n makes n47n0 divisible by 36?

4. A father divides all his money among his four sons so that one son gets $\frac{1}{2}$; a second son, $\frac{1}{4}$; a third son, $\frac{1}{5}$; and the fourth son, \$10 000. How much money did the father have?

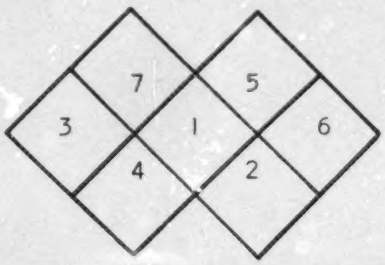
5. A pool is being filled with two hoses. One hose can fill the pool in 24 hours and the other in 36 hours. How many hours will it take to fill the pool if they are used at the same time?

6. Find each sum:
   A) $1 + 3 + 5 = ?$
   B) $1 + 3 + 5 + 7 = ?$
   C) $1 + 3 + 5 + 7 + 9 = ?$
   D) $1 + 3 + 5 + 7 + 9 + 11 = ?$
   E) $1 + 3 + 5 + 7 + 9 + 11 + 13 = ?$
   F) $1 + 3 + 5 + 7 + ... + 289 + 291 = ?$
   G) $1 + 3 + 5 + 7 + 9 + ... + x = ?$

PROBLEM SHEET 9.2

ANSWERS AND SELECTED SOLUTIONS

1. There are a variety of possible solutions, one of which is shown. All solutions will have the "1" in the centre.



2. $0.\overline{012345679}$

3. If the number is divisible by 36, then it must be divisible by 9 and 4. The sum of the digits must be a multiple of 9 and the last two digits must be divisible by 4. Therefore, n = 8.

4. The fourth son gets $1 - (\frac{1}{2} + \frac{1}{4} + \frac{1}{5}) = \frac{1}{20}$ of the estate.
Therefore, the father has \$10 000 $\div \frac{1}{20}$ = \$200 000.

5. In one hour they will fill $\frac{1}{24} + \frac{1}{36} = \frac{3 + 2}{72} = \frac{5}{72}$ of the pool.

Therefore, they will take $\frac{72}{5} = 14\frac{2}{5}$ hours to fill the pool.

6. A) 9
B) 16
C) 25
D) 36
E) $\left(\frac{13 + 1}{2}\right)^2 = 49$
F) $\left(\frac{291 + 1}{2}\right)^2 = 21\ 316$
G) $\left(\frac{x + 1}{2}\right)^2$

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PROBLEM SHEET 9.3

1. A salesman sold $5288 worth of goods in one week. He was paid $12\frac{1}{2}\%$ commission on his sales, plus $256.00 as a regular weekly wage. What would be his total earnings for the week?

2. How many positive integers less than a thousand have the sum of their digits equal to four?

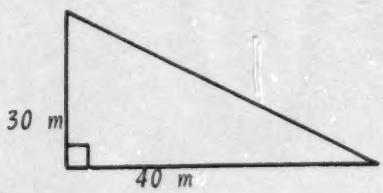
3. List the ways you can make a sum of $2x - 5$ from the entries in the square below.

$x + 1$	$4x + 8$	$-x + 5$
$-x - 17$	$x - 6$	$3x - 10$
$-6 - x$	$-(x + 7)$	$4 - x$

4. If $a * b = ab - 1$ and $a \nabla b = a - b$, what is the value of $4 * (6 \nabla 8)$?

5. Determine which is the greater quantity: 2^{80} or 4^{40}.

6. A plot of ground is in the shape of a right triangle with sides of length 30 m and 40 m. A fence is to be built around the piece of ground with fence posts 5 m apart. How many fence posts are needed?



PROBLEM SHEET 9.3

ANSWERS AND SELECTED SOLUTIONS

1. $917.00

2. 4, 13, 22, 31, 40, 103, 112, 121, 130, 202, 211, 220, 301, 310, 400.
   There are 15 integers.

3. $(x + 1) + (x - 6) = 2x - 5$
   $(3x - 10) + (-x + 5) = 2x - 5$
   $(4x + 8) + (-x - 17) + (4 - x) = 2x - 5$
   $-(x + 7) + (4x + 8) + (-6 - x) = 2x - 5$

4. $4 * (6 * 8) = 4 * (6 - 8)$
   $= 4 * (-2)$
   $= 4 (-2) - 1$
   $= -9$

5. $2^{80} = 2^{4 \times 20}$ $\qquad$ $3^{60} = 3^{3 \times 20}$
   $= 16^{20}$ $\qquad$ $= 27^{20}$
   Since $27 > 16$, then 3^{60} is larger than 2^{80}.

6. Perimeter = 30 m + 40 m + 50 m = 120 m
   Number of posts = $\frac{120}{5} = 24$

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PROBLEM SHEET 9.4

1. There are 100 cards numbered consecutively from 1 to 100. How many ways can you pick 2 cards whose sum is 50?

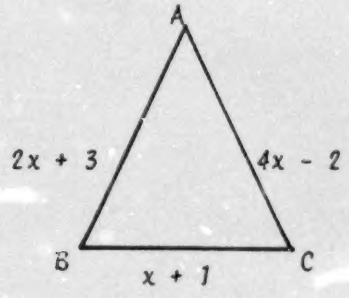
2. On a $100 purchase, Sheldon was offered 3 successive discounts of 5%, 10%, and 20%. In what order should Sheldon take the discounts to give him the largest discount?

3. Find the number in which the sum of the number, its square, and its square root is 93.

4. A watch is stopped for 20 minutes every hour on the hour. How many actual hours elapse between the time at which the watch shows 12 noon and the time at which it shows 12 midnight?

5. If $x + y = -3$, $y + z = -4$, and $x + z = 3$, find the value of $x + y + z$.

6. In isosceles triangle ABC with $\overline{AB} = \overline{AC}$, find BC if $AB = 2x + 3$, $AC = 4x - 2$ and $BC = x + 1$.



PROBLEM SHEET 9.4

ANSWERS AND SELECTED SOLUTIONS

1. 24 ways

2. Any order gives the same result.

3. 9

4. $\frac{12 \times 20}{60} + 12 = 16$ hours

5. Therefore, adding the sum of the three expressions given in the problem will give us $2x + 2y + 2z = -3 + (-4) + 3$

$$= -4$$

Therefore $(x + y + z) = \frac{-4}{2}$

$$= -2$$

6. If AB = BC, then

$$2x + 3 = 4x - 2$$

$$2x - 4x = -2 - 3$$

$$-2x = -5$$

$$x = \frac{5}{2}$$

$$BC = x + 1$$

$$= \frac{5}{2} + 1$$

$$= 3\frac{1}{2}$$

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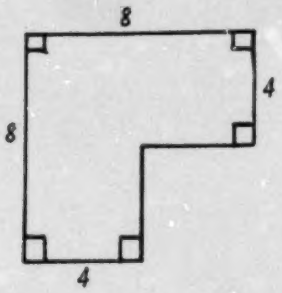
PROBLEM SHEET 9.5

1. Ms Sedley worked 8 hours a day for six days a week and received no overtime pay. What would her net pay be if she received $12.35 per hour and 25% of the total earnings are deducted for income tax?

2. What numbers added to themselves give the same number as their square?

3. If $a + b + c = 6$, $a + b = -4$, and $a + c = 9$, find b.

4. How can the figure below be divided into four figures which are equal in area and have the same shape as the original figure?



5. A pizza and a coke together cost $6.00. If the pizza costs $5.00 more than the coke, how much does the coke cost?

6. A computer is programmed to scan the digits of the integers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ... To this point it has scanned 15 digits. It continues until it has scanned 1000 digits. What is the integer it is scanning at that point?

PROBLEM SHEET 9.5

ANSWERS AND SELECTED SOLUTIONS

1. $444.60

2. 0 and 2

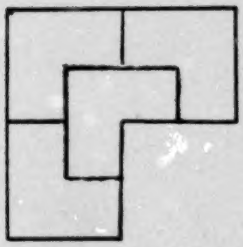
3. If $a + c = 9$

   then $(a + c) + b = 6$

   $a + b = 6$

   $b = -3$

   $a + b = -4$ is not required for the solution.

4. 

5. Coke costs $0.50.

6.

Integers scanned	Number of digits scanned	Total sum of digits scanned
1 - 9	9	9
10 - 99	90 x 2 = 180	189

For the next 900 integers the computer will scan 3 digits for each integer. There are 1000 - 189 = 811 digits left to scan.

The computer is scanning the ($811 \div 3 = 270\frac{1}{3}$) 271st three digit integer.

The integer when the computer is scanning the 1000th digit is 271 + 99 = 370.

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PROBLEM SHEET 9.6

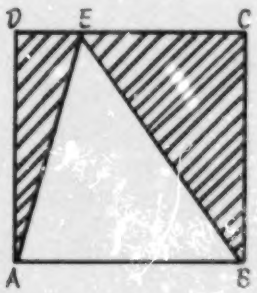
1. How many 3-digit numbers ending in 25 are a perfect square? Write the numbers.

2. A certain number of pennies and twice as many nickels are worth 77 cents. Find the number of pennies and the number of nickels.

3. A train one kilometre long is travelling at a steady speed of 30 km/h. It enters a tunnel one kilometre long at 1:00 p.m. At what time does the rear of the train emerge from the tunnel?

4. If the average of $3 + x$, 4^2, 12 and $7x + 1$ is $6x$, find the value of x.

5. Square ABCD has side x units. Find the area of the shaded region in terms of x.



6. The integers greater than one are arranged in five columns as follows:

	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	

In which column will 1000 fall?

PROBLEM SHEET 9.6

ANSWERS AND SELECTED SOLUTIONS

1. There are two: $225 = 15^2$ and $625 = 25^2$.

2. $1 \cdot x + 2(5 \cdot x) = 77$ where x = number of pennies

   $x = 7$

   There are 7 pennies and 14 nickels.

3. After travelling 2 km the train is out of the tunnel.

$$t = \frac{\text{distance}}{\text{rate}}$$

$$t = \frac{2}{30}$$

$$= \frac{1}{15} \text{ of an hour}$$

   The rear of the train emerges at 1:04 p.m.

4. $x = 2$

5. Area of $\square ABCD = x^2$

   Area of $\triangle AEB = \frac{1}{2}$ bh

   $= \frac{1}{2}(x)(x)$

   Area of shaded region $= \frac{x^2}{2}$

6.

	2	3	4	5
9	(8)	7	6	
	10	11	12	13
17	(16)	15	14	
	18	19	20	21
25	(24)	23	22	

NOTE: All numbers divisible by 8 are found in the second column. Since 1000 is divisible by 8, then 1000 is in column 2.

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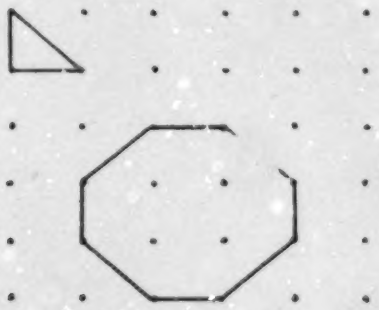
PROBLEM SHEET 9.7

1. How many minutes will it take to cut a strip of cloth 50 metres long into strips 1 metre long, if each cut takes 1 minute?

2. The area of the triangle is 2 cm^2. What is the area of the octagon?



3. The sum of the ages of 2 children is 14 years. How long will it be until the sum of their ages is 22 years?

4. A coin collector went to the bank to exchange a $10 bill for nickels, dimes, and quarters, and received an equal number of each type of coin. How many of each type did the coin collector receive?

5. Mr. Growgo has a 20 m x 20 m square garden. In the centre is a 10 m x 10 m square patch which he cannot use. If he divides the usable space into four congruent rectangular patches, what are the dimensions of each patch?

6. The lengths of the sides of an isosceles triangle are whole numbers in the ratio of 6:5. If the perimeter of the triangle is 32 cm, find the area of the triangle.

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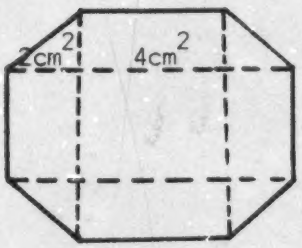
PROBLEM SHEET 9.7

ANSWERS AND SELECTED SOLUTIONS

1. 49 minutes

2. The area of the octagon is found by partitioning the figure as shown.

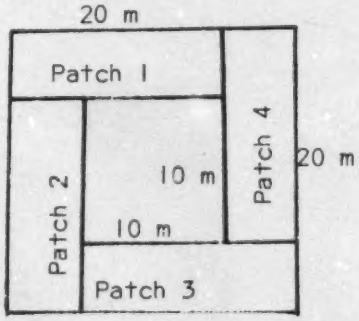


Area = 28 cm^2

3. $\frac{22 - 14}{2}$ = 4 years

4. 25 coins of each type

5.



Dimensions of each patch are 15 m x 5 m

6. There are two possibilities for the triangle's shape. Its sides are either of 6 : 5: 5 or 5: 6: 6. Since 5 + 6 + 6 = 17 and 17 is not a multiple in the ratio of 32 the sides must be in the ratio of 6 : 5 : 5. Since 6 + 5 + 5 = 16 the sides must measure 12, 10 and 10 respectively. Find the altitude x by using the Pythagorean theorem.

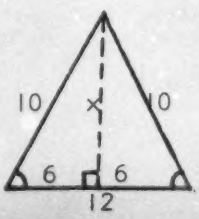
$6^2 + x = 10^2$

$x^2 = 10^2 - 6^2$

$x^2 = 64$

$x = 8$

Area = $\frac{12 \times 8}{2}$ = 48 sq.units



PROBLEM SHEET 9.8

1. The owner of a car drove 12 240 kilometres last year, and his car averaged 10 litres per 100 km. How much money did he spend for gasoline if gasoline costs 44¢ a litre?

2. How many quadrilaterals can be drawn using any 4 of the 5 points on the circle as vertices of the quadrilateral?

3. The dimensions of a rectangle are 6 cm by 24 cm. Find the dimensions of a square with the same area.

4. Mr. Smith sold 3 pieces of furniture. He sold the second piece for twice as much as the first and he sold the third piece for $13.00 more than the second. He collected $888.00 altogether. How much did each piece cost?

5. What real number values for x satisfy $x^2 < x$?

6. For a class of 10 students, the teacher posted the marks as follows: 72%, 83%, 63%, 88%, 72%, 63%, 97%, 52%. The mode, median and mean marks were all the same. What marks did she forget to post?

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PROBLEM SHEET 9.8

ANSWERS AND SELECTED SOLUTIONS

1. Amount of gas used is $\frac{12\ 240}{100} \times 10 = 1224$ litres.
   Cost is $1\ 224 \times 0.44 = \$538.56$.

2. 5 quadrilaterals

3. 12 cm x 12 cm

4. \$175.00, \$350.00, \$363.00

5. All real numbers between zero and one.

6. The mode must be one of the marks already listed. (This is so since we already have some marks which occur twice, eg., 63% and 72%, and we have only 2 missing marks).

   Ordering of the numbers from smallest to largest gives: 52 63 63 72 72 83 88 97.

   In order to satisfy the conditions of a median (middle marks), one of the missing marks has to be 72.

   To satisfy the requirement that the average must be the same number as the median and the mode, find the second missing number by setting up a number sentence which determines the average, as follows:

$$\frac{x + 72 + 72 + 83 + 63 + 88 + 72 + 63 + 97 + 52}{10} = 72$$

$$x = 58$$

   The marks are 72 and 58.

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PROBLEM SHEET 9.9

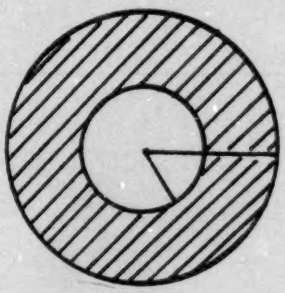
1. A man sold $3840 worth of goods in a 40 hour work week. If he is paid $12.50 per hour and $4\frac{1}{2}\%$ commission on his sales, what would be his total earnings?

2. a) Solve for x: $2^x = 8$

   b) Solve for n: $2^{(3n - 1)} = 32$

3. At a party, exactly 15 people ate hot dogs and exactly 12 people ate hamburgers. Ten of these people ate both. Three people ate neither. Find the number of people at the party.

4. Two concentric circles are shown in the diagram. The radius of the outer circle is 10 cm and the radius of the inner circle is 5 cm. Find the area of the shaded region. (Use $\pi = 3.14$.)



5. The probability of flipping a coin and getting "heads" is 1 out of 2.
   a) What is the probability of flipping 4 tails in a row?
   b) What is the probability of flipping 10 heads in a row?

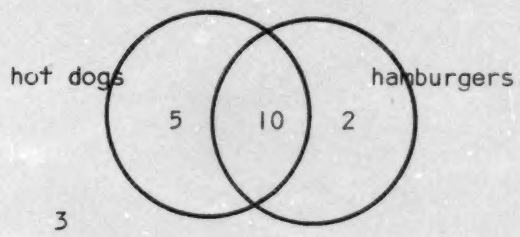
6. It takes 753 digits to number the pages of a book consecutively (i.e., page 106 has 3 digits). How many pages are there in the book?

PROBLEM SHEET 9.9

ANSWERS AND SELECTED SOLUTIONS

1. $(40 \times \$12.50) + (3840 \times 0.045) = 672.80$.
   Total earnings are \$672.80.

2. a) $x = 3$
   b) $2^{3n-1} = 32 \qquad 2^{3n-1} = 2^5$
   $3n-1 = 5 \qquad n = 2$

3. 

   The number of people at the party is $5 + 10 + 2 + 3 = \underline{20}$

4. Outer circle area $= 3.14 \times 10 \times 10$
   $= 314 \text{ cm}^2$
   Inner circle area $= 3.14 \times 5 \times 5$
   $= 78.5 \text{ cm}^2$
   Shaded area $= 314 - 78.5$
   235.5 cm^2

5. a) $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$ out of 16
   b) $\left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$ or 1 out of 1024

6. The first 9 pages (pages 1-9) use 9 digits.
   The next 90 pages (pages 10-99) use 180 digits.
   There are still (753 - 189) or 564 digits remaining.
   Starting at page 100 three digits are required per page. Therefore,
   $564 \div 3 = 188$ more pages can be numbered.
   There are $9 + 90 + 188$ or 287 pages.

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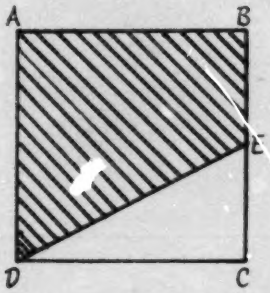
PROBLEM SHEET 9.10

1. If a plane travels 900 km in an hour and a half, find its speed in km/h.

2. There are 120 apple and peach trees in an orchard. There are $\frac{2}{3}$ as many peach trees as apple trees. How many trees of each kind are there?

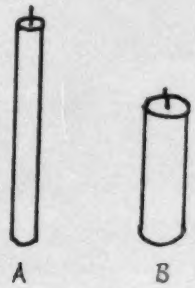
3. How many pairs of values are there for x and y which satisfy the equation $2x + 3y = 37$ if positive integers are used?

4. ABCD is a square. E is the mid-point of $\overline{BC}$. AB = 6. Find the area of the shaded region.



5. Find a two digit number which is equal to twice the sum of its digits.

6. A represents a candle 22 cm tall. It will burn completely in 2 hours. B represents a candle 15 cm tall. It is a slower burning candle and will burn completely in 3 hours. Both candles are lighted at noon. At what time are they the same length?



PROBLEM SHEET 9.10

ANSWERS AND SELECTED SOLUTIONS

1. 600 km/h.

2. 72 apple trees
   48 peach trees

3. (17, 1), (11, 5), (5, 9), (14, 3), (8, 7), (2, 11)

4. Area of square = 6 x 6 sq. units.
   Area $\triangle$CDE = $\frac{6 \times 3}{2}$
   = 9 sq. units
   Area of shaded area
   = 36 - 9
   = 27 sq. units

5. $10x + y = 2(x + y)$
   $10x + y = 2x + 2y$
   $8x = y$
   The only digits which satisfy $8x = y$ are $x = 1$ and $y = 8$.
   The only two digit number is 18.

6. Let t be the number of hours required for the candles to be of the same length. Since the burning speed of candle A is 11 cm/h and that of B is 5 cm/h, then:
   $$22 - 11t = 15 - 5t$$
   $$7 = 6t$$
   $$1\frac{1}{6} \text{ h} = t$$
   The candles are the same length at 1:10 p.m.

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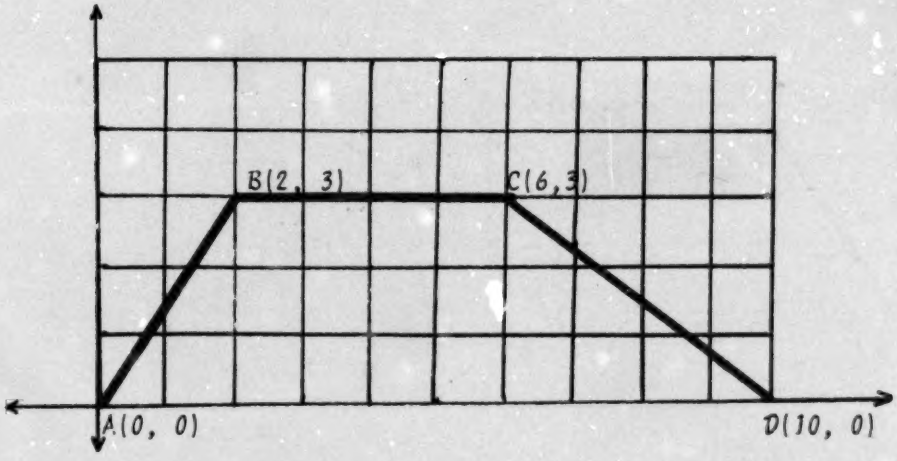
PROBLEM SHEET 9.11

1. Megan put an equal number of beetles and spiders into a bottle. If there were a total of 168 legs, how many beetles and how many spiders did she put in the bottle? (Spiders have 8 legs; beetles have 6 legs.)

2. Find all real number values of x which satisfy $\frac{3}{x} \leq 1$.

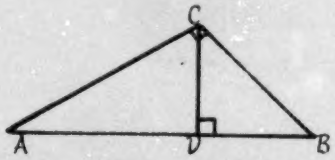
3. Find the area of quadrilateral ABCD if each square has a side of 1 cm.



4. There are 930 people watching a musical at Rainbow Stage. If there are 490 males and 260 children, of whom 150 are girls, how many adult females are watching the musical?

5. An open box is constructed from a rectangular sheet of metal 20 cm by 14 cm. Pieces are cut out of the corners so that when the sides are folded the box will be 2 cm deep. What is the volume of the box?

6. Triangle ACB is a right triangle with the right angle at C. $\overline{CD}$ is perpendicular to $\overline{AB}$. If AC = 8 and BC = 6, find the lengths of $\overline{AB}$ and $\overline{CD}$.



PROBLEM SHEET 9.11

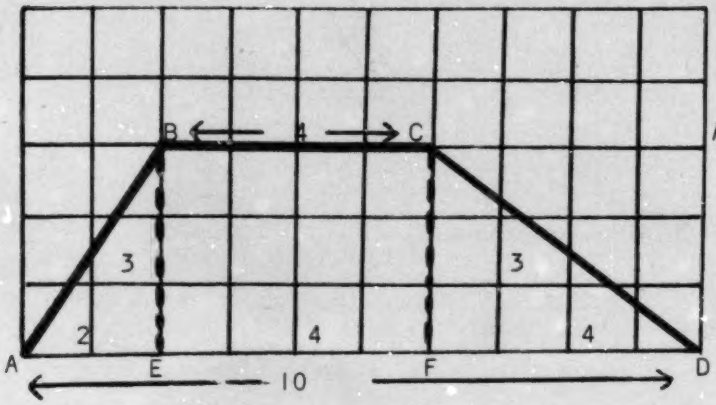
ANSWERS AND SELECTED SOLUTIONS

1. Let x = number of beetles and x = number of spiders.

   $8x + 6x = 168; \quad x = 12$

   There are 12 beetles and 12 spiders.

2. $x \geq 3$ and $x < 0$.

3. 

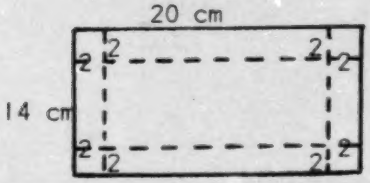
   A = $\triangle ABC$

   = 3 + 12 + 6

   = 21 sq units

4.

Men 380	Boys 110
Women 290	Girls 150

5. 

   Volume = (16) (10) (2)

   = 320 cm^3

6. With the Pythagorean theorem, AB = 10

   Area of $\triangle ABC = \frac{1}{2}$ (AC) (BC) = $\frac{1}{2}$ (8) (6) = 24

   Area of $\triangle ABC = \frac{1}{2}$ (AB) (CD)

   $24 = \frac{1}{2}$ (10) (CD)

   $CD = \frac{24}{5}$

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PROBLEM SHEET 9.12

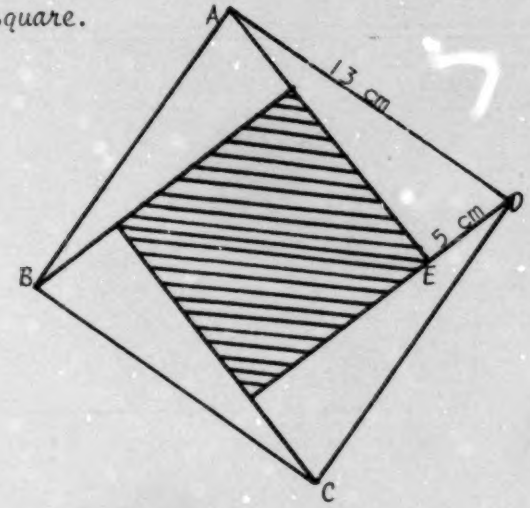
1. A house is worth \$92 000.00 and its contents worth \$40 000.00. What will it cost to insure both the house and the contents for 74% of their value if the premium is \$2.40 per \$1000.00 insured?

2. If the reciprocal of $x + 1$ is $x - 1$, find x.

3. Find the area of parallelogram ABCD with vertices A(-3,2); B(3,2); C(6,4); and D(0,4).

4. When a teacher gave 3 sheets of paper to each student, she had 23 sheets left. When she gave 4 sheets of paper to each student, she had 5 sheets left.
   a) How many students were in the class?
   b) How many sheets of paper did the teacher have?

5. Find the area of the shaded region if AD = 13 cm and DE = 5 cm. ABCD is a square.



6. If x, y, and z are integers, and if $\frac{1}{x} + \frac{1}{y} = z$, find all possible values for z.

PROBLEM SHEET 9.12

ANSWERS AND SELECTED SOLUTIONS

1. $234.43

2. $\frac{1}{x+1} = x - 1$

$1 = (x - 1)(x + 1)$

$1 = x^2 - 1$

$2 = x^2$

$\pm\sqrt{2} = x$

3. $A = b \times h$

$= 6 \times 2$

$= 12$ sq. units

4.

Number of sheets given to each student	Number of sheets left
3	23
4	5

a) Since giving 1 additional sheet to each student reduces the surplus of sheets by 18, the number of students is 18.

b) Number of sheets the teacher had = $18 \times 3 + 23 = 77$

5. The length of $\overline{AE}$, calculated using the Pythagorean theorem, is 12 cm. The shaded square's side will thus be 12 - 5 or 7 cm. The shaded area is therefore 7×7 or 49 cm^2.

6. $z = 0, \pm 1, \pm 2$

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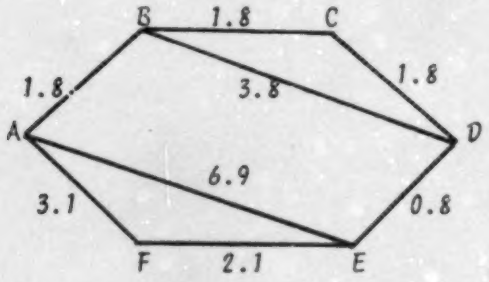
CALCULATOR PROBLEM SHEET 9.13

1. The distance between any two points is calculated by multiplying the branch numbers.
   Find the shortest path from A to D.



2. The average of three numbers is 0.5735. Two of the numbers are $\frac{16}{125}$ and $\frac{5}{16}$. Find the third number.

3. Solve the following:

   a) $6^2 - 5^2$

   b) $(56)^2-(45)^2$

   c) $(556)^2-(445)^2$

   d) $(5556)^2-(4445)^2$

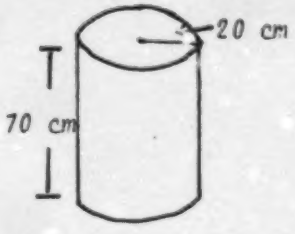
   e) Using the pattern established in (a) through (d), solve $(5555556)^2-(4444445)^2$.

4. A bicycle tire lasts for 3678 km and its radius is 40 cm.
   a) How many rotations does the tire make during its lifetime? (Use $\pi = 3.14$.)
   b) If 0.6 cm of tread is worn off during the life of the tire, how much tread is lost during each rotation?

CALCULATOR PROBLEM SHEET 9.13

5. Fermat's Conjecture (1640) states that every prime number of the form $(4n + 1)$ is the sum of two squares in exactly one way.
For example: if $n = 3$ then $4n + 1 = 13 = 2^2 + 3^2$
if $n = 7$ then $4n + 1 = 29 = 2^2 + 5^2$
What are the two squares if:
   a) $n = 9$
   b) $n = 24$
   c) $n = 78$

6. Grottens Hamburgers have sold 19 564 000 hamburgers thus far. The relish for the hamburgers comes in cylinder containers as shown:



   a) If 0.7 cm^3 of relish is used on each hamburger and if Grottens were to stack the cylinders on top of each other, how high would the stack be in metres? (Use $\pi = 3.14$.)
   b) How does this compare to the tallest structure in your community?

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CALCULATOR PROBLEM SHEET 9.13

ANSWERS AND SELECTED SOLUTIONS

1. A to F to E to D or $3.1 \times 2.1 \times 0.8 = 5.208$

2. $0.5735 \times 3 - 0.128 - 0.3125 = 1.28$.

3. a) $36 - 25 = 11$
   b) $3136 - 2025 = 1111$
   c) $309\ 136 - 198\ 025 = 111\ 111$
   d) $30\ 869\ 136 - 19\ 758\ 025 = 11\ 111\ 111$
   e) $11\ 111\ 111\ 111\ 111$

4. a) $C = 2\pi r$
   $= 2 \times 3.14 \times 40$
   $= 251.2$ cm
   $= 0.002512$ km
   $3768 \div 0.002\ 512 = 1\ 500\ 000$ rotations
   b) $0.6 \div 1\ 500\ 000 = 0.000\ 000\ 4$ cm

5. a) $4n + 1 = 4 \times 9 + 1$
   $= 37$ $\qquad 37 = 6^2 + 1^2$
   b) $4n + 1 = 4 \times 24 + 1$
   $= 97$ $\qquad 97 = 9^2 + 4^2$
   c) $4n + 1 = 4 \times 78 + 1$
   $= 313$ $\qquad 313 = 12^2 + 13^2$

6. a) Volume of cylinder $= 20^2 \times 3.14 \times 70$
   $= 87\ 920\ cm^3$
   Volume of relish used $= 0.7 \times 19\ 564\ 000$
   $= 13\ 694\ 800$
   Number of cylinders needed $= 13\ 694\ 800 \div 87\ 920$
   $= 155.76\ 433$ or 156
   Height of cylinders $= 156 \times 70$ cm
   $= 10\ 920$ cm or 109.20 m
   b) Answers will vary with location.

PROBLEM SHEET 9.14 - SUPERSTAR

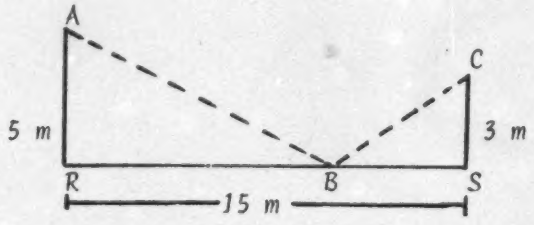
1. If $x + y = 5$ and $xy = 13$, evaluate $\frac{1}{x} + \frac{1}{y}$.

2. In the sequence, 1, 3, 5, 2, 4, 6, 3, 5, 7, 4, 6, 8, ..., find the sum of the first sixty terms.

3. Find the area of the triangle enclosed by the lines $y = x$, $x + y = 6$, and the x axis.

4. If $a^2 + b^2 = 15$ and $ab = 6$, find the value of $(a - b)^2$.

5. A hockey player shoots the puck from position A. It bounces off the sideboard at B and reaches the player at C. How far did the puck travel from position A to position C?



6. Five cross-country skiers, each having a number on his back from 1 to 5 (no duplicates), cross the tape at the end of a race, the first at 9:01, and the others at intervals of one minute apart. There are no ties. From the following facts, determine the order in which the runners came in. In each statement, "time" means the number of minutes after 9:00.
   1) The highest total of a runner's number and his time is 9 and occurs just once.
   2) The lowest total of a runner's number and his time is 3 and occurs just once.
   3) The total 7, of a runner's number and time, occurs just twice.
   4) The winner's number exceeds that of the second to come in.

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PROBLEM SHEET 9.14 - SUPERSTAR

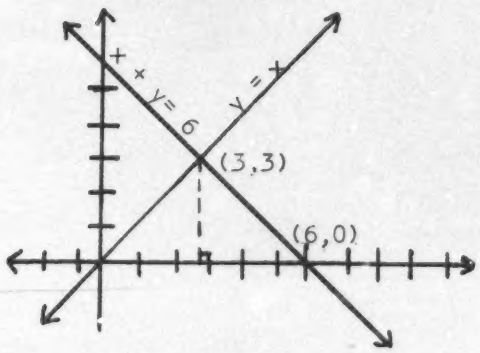
ANSWERS AND SELECTED SOLUTIONS

1. $\frac{1}{x} + \frac{1}{y} = \frac{x + y}{xy} = \frac{5}{13}$

2. (1, 3, 5), (2, 4, 6), (3, 5, 7), (4, 6, 8), ..., (19, 21, 23), (20, 22, 24)
If the sequence is grouped in threes, then there will be $60 \div 3 = 20$ such groups. The first number of each group yields the sequence 1, 2, 3, ..., 20. The second number of each group yields the sequence 3, 4, 5, ..., 22. The third number of each group yields the sequence 5, 6, 7, ..., 24.

$1 + 2 + 3 + \ldots + 20 = 210$
$3 + 4 + 5 + \ldots + 22 = 250$
$5 + 6 + 7 + \ldots + 24 = 290$
Sum = 750

3.



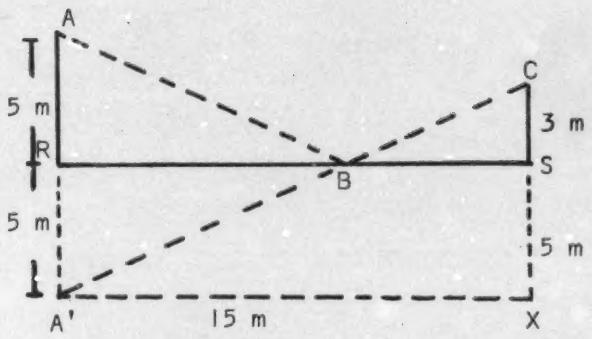
Area $= \frac{1}{2}(6)(3)$
$= 9$ square units

4. $(a - b)^2 = a^2 - 2ab + b^2$
$= (a^2 + b^2) - 2ab$
$= 15 - 2(6)$
$= 3$

PROBLEM SHEET 9.14 - SUPERSTAR

ANSWERS AND SELECTED SOLUTIONS

5.



Reflect A in the line $\overline{RS}$ to A'

AB = A'B

Extend $\overline{CS}$ to X so that SX = 5 m.

Join A' to X.

A'CX is a right-angled triangle.

$(A'C)^2 = (A'X)^2 + (XC)^2$

$(A'C)^2 = 225 + 64$

$(A'C)^2 = 289$

$A'C = 17$

The puck travels 17 m from position A to C.

6. One possible approach to the solution is to set up a table and use guess and check. The order of runners is 3, 1, 4, 5, 2.

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